PROBLEMS

CONNECTED WITH

FLOOD DRAINAGE.

A SERIES OF SIX LECTURES DELIVERED AT THE SIBPUR ENGINEERING COLLEGE

BY

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ADDENDA AND CORRIGENDA.

Page 10.—4th line after equation (4)

For—" ... when $\frac{d}{h'}$ is as large ... "

Read—" ... when $\frac{d'}{h'}$ is as large ... "

Page 15.—last line

For—" ... the ratio $\frac{h_1}{h\square_o}$ ... "

Read—" ... the ratio $\frac{h_1}{h_o}$ ... "

Page 16.—6th line below equation (21)

On the right-hand side of the equation

For—" $\frac{dh}{h}$ "

Read—" $\frac{dh}{h^{t}}$ "

Page 20.—In the 2nd line of Table IX

For—" $\frac{q}{q_o}$ "

Read—" $\frac{q_m}{q_o}$ "

Page 22.—1st line below equation (34)

For—" ... two ratios $\frac{h}{h_o}$ ... "

Read—" ... two ratios $\frac{h_1}{h_o}$ ... "

Page 33.—4th line from bottom

On the right-hand side of the equation

For—" $-\frac{\rho d\rho}{\rho - r}$ "

Read—" $-\int_{\rho}^{1.01r} \frac{\rho d\rho}{\rho - r}$ "

Page 37.—Equation (63)

For—" $q = C_w \frac{2}{3} cb \sqrt{2g}\ h$ "

Read—" $q = C_w \frac{2}{3} cb \sqrt{2g}\ h^{t}$ "

Page 42.—2nd line after equation (73)

For—" of c, in this case ... "

Read—" of C, in this case ... "

Page 54.—Left-hand side of equation (87)

For—" $x = ...$ "

Read—" $ix = ...$ "

Page 83.—13th line of paragraph 2

For—" a rate of a flow ... "

Read—" a rate of flow ... "

Page 86.—5th line from bottom

For—" Total ... 7·00 × 4,753 ... ... = 33,343 "

Read—" Total ... 7·00 × 4,763 ... ... = 33,343 "

PROBLEMS CONNECTED WITH FLOOD-DRAINAGE.

I.—INTRODUCTION.

The subject which I propose to deal with in this course of lectures is one for which I have had some difficulty in finding a suitable title. The title which I have selected as being perhaps more nearly applicable than any other I could think of is "Problems connected with flood-drainage;" and the reason for its selection is mainly that most of the problems for which I have endeavoured to find practicable solutions have presented themselves to me while engaged in the attempt to unravel some of the complicated conditions which confront the designer of engineering works for the drainage of large flooded tracts of country, and becoming specially complicated when the flood-water has to be discharged into tidal waters. My subject, then, consists of a collection of problems, arising out of the conditions I have mentioned; but I wish to avoid implying that they are only applicable to questions of flood drainage, because some of them apply to the conditions met with in designing irrigation works, or, in fact, wherever we have to deal with and control the flow of large quantities of water. Now the first thing that the Engineer has to consider is the cheapest and most efficient means of conveying water in large quantities; and to find the method by which this can be done he has only to use his eyes and take a lesson from Nature. In every country Nature is constantly at work transporting water from the high lands of the interior to the ocean by means of streams and rivers, varying in size from the tiny rivulet to mighty rivers like the Amazon, the Mississipi, the Ganges or the Mahanadi. This last-mentioned river, when in flood, pours into the sea a volume of about $1\frac{1}{2}$ million cubic feet in every second of time. It will perhaps enable you to realise better what that figure implies, when I say that if you were to stand on the bank of the Mahanadi at Cuttack in full flood, during one hour there would flow past you enough water to cover the whole of the Calcutta maidan to nearly the height of the Ochterlony monument. Thus the general principle of conveying water in open channels is simple enough, and the Engineer has no difficulty in deciding on the construction of an artificial river, or open channel, as the most economical method. It is in the way he sets about his work that he has to depart from Nature's methods. A natural river is formed simply by the erosive action of the water, as it flows over the land, down towards the sea. The water thus scoops out a channel for itself, in the process of time, and the channel is of course just big enough to carry the quantity of water which has scooped it out, generally speaking. The Amazon would not have so large a bed if the greatest quantity of water that flowed down it were only as much as is discharged by the Thames; and conversely the Thames would be very much wider and deeper if it had been scooped out by a volume similar to that discharged by the Amazon. The Engineer, however, cannot, for obvious reasons, follow this convenient plan of just letting the water run, and allowing the channel to adjust its own size. He has first of all to find out what is the quantity of water to be conveyed, and then he has to determine beforehand, by calculation, exactly what width, depth, and slope of channel will suffice to convey it.

But his work does not end when he has designed his channel. He also has to devise artificial means of regulating and controlling the flow of water in his channel, so as to attain his end, whether drainage or irrigation or water-supply, by the most efficient means; and with this object he has to construct masonry works, such as sluices, weirs, regulators, falls, escapes, and so on.

To take an example. Let us suppose that an irrigation canal has to be constructed, and that the earth in the locality is of such a nature that it is liable to be cut away by water flowing faster than a certain rate, so that the speed of the current in the canal must be kept below that rate. Now you know that the velocity of flow depends largely upon the fall or slope of the canal in the direction of flow; and we will suppose that the Engineer finds by

calculation that the limiting velocity in his canal will be attained by a slope of 4 feet in the mile. But the levels he has taken show that the country through which his canal has to be made has a natural slope of 8 feet in a mile. In such a case Nature's method would be to reduce the slope of her river by increasing its length, *i.e.*, by making it crooked. If it was originally straight, the banks would be cut away, on one side or the other, by the high velocity of the current, until the course of the river became so crooked that its length was about doubled, and thus the slope, and consequently the velocity, was reduced to the limit at which cutting would cease. Thus, if two places, A and B, (fig. 1) were 20 miles apart, measured in a straight line, and the difference of level was 160 feet, then the slope would be at the rate of ($\frac{160}{20}=$) 8 feet per mile; but if the canal were doubled in length, the slope would be ($\frac{160}{40}=$) 4 feet per mile, which is the limit we assumed just now at which cutting would cease. This is one of the reasons why rivers are crooked, and the fact is of importance to the Engineer who has to deal with the training of rivers and the maintenance of embankments. To return to our canal, it is manifestly out of the question to design a canal of double the necessary length, and the Engineer has to again depart from Nature's methods and devise some other means of reducing the velocity.

Supposing the depth of the water in the canal is to be 8 feet, and at the point A (fig. 2) the surface of the water is level with the ground, so that the bed of the canal is 8 feet below ground-level, *i.e.*, the depth of excavation is 8 feet. Now, if we make the bed of the canal slope at the rate of only 4 feet per mile, while the ground slopes at the rate of 8 feet, it is clear that at the end of a mile the bed of the canal will be only 4 feet below ground level, and at the end of 2 miles the bed will be at the same level as the ground, while the surface of the water will be 8 feet above the ground. In such a case the banks of the canal would have to be built up with earth taken from the land outside, to a height sufficient to hold up the water. At this rate the surface of the water at the end of 4 miles would be no less than 16 feet above ground level, a height which would in ordinary cases be quite inadmissible. You will now see that, if we wish to grade the bed of the canal at a flatter slope than the ground, it is necessary to resort to the construction of artificial waterfalls at intervals, each of which causes a sudden drop in the bed of the canal, which is thus again brought below ground level, enabling the same method of grading to be repeated for another length; and thus the flat slope can be continued to any distance, proceeding by a series of steps, as shown in fig. (2).

Here, again, we are only copying the natural waterfalls which may be seen in hilly country; but where we go beyond Nature is in applying to the soft alluvial soil of the plains the methods which she only uses in steep and rocky ground.

Another example of an artificial regulating work is a sluice. In the case of an irrigation canal it is obvious that some means of regulating the admission of water into the canal is necessary, so as to supply water only at such times as it is required for the crops. This is done by means of a sluice, which in this case consists practically of a masonry dam across the head of the canal, containing numerous openings, which are fitted with iron or wooden shutters, which can be opened or closed by means of suitable mechanism. But sluices are also very often necessary at the outfalls of drainage channels, where they serve a different purpose. If the drainage channel discharged into a river whose level never varied, and which was not liable to flood, a sluice might not be required. If, however, the river was liable to floods, then in flood time the level of the river might rise above the level of the country to be drained, and then the channel, instead of draining the flood water off the country, would actually bring the river water on to the land, and increase the flooding. Besides, when rivers are in flood their waters are usually heavily charged with silt, which would enter the channel and, being deposited there, reduce its section, and cause heavy expenditure in clearing it out again. When this is liable to occur it is necessary to provide a sluice or regulator, which would be closed so as to prevent water from the river entering the drainage channel in times of flood, and would be opened when the river fell again, so as to allow the flood water to flow from the channel into the river, and thus permit drainage to go on. The provision of a sluice is more than

ever necessary when the drainage channel discharges into a tidal river heavily laden with silt, like the Hooghly. Drainage can only occur during low tide, and if the mouth of the channel is left open when the tide rises, the muddy water from the river enters the channel, which is thus filled and emptied twice every day, and much of the silt from the river is deposited in it. In course of time the channel becomes reduced in size, and may at length disappear completely, leaving no outlet for the rain-water which falls on the land. A very good example of this may be seen now in the tract of country lying around and near Magra Hât in the 24-Parganas, about 25 miles south of Calcutta. Formerly much of this country was drained by natural channels into the Diamond Harbour Creek, but unfortunately when Nature made these channels she neglected to provide them with sluices at their mouths, with the result that all the channels have silted up so much from the mud brought in from the Hooghly with every tide, that the rain-water which falls on the low-lying land has no outlet whatever, and remains on the land until it evaporates. In the year 1901 many thousands of acres of rice perished from this cause, and the loss to the cultivators and zamindars was estimated at over 80 lakhs of rupees. Another use to which sluices can be put is shown in some of the large drainage-works near Howrah. Irrigation is carried on at certain times of the year, and the sluices are used either to let out rain-water from the fields, or to admit water from the Hooghly to the fields; they serve, in fact, to regulate the depth of the water on the fields according to the requirements of the crops, in addition to the purposes of excluding silt from the channels and passing out drainage water.

I have used the term "regulator," and we have seen that sluices are used for the purpose of regulating water, and thus in the general sense of the word serve as regulators; but the word "regulator" is commonly used to denote a particular kind of masonry work, somewhat different from a sluice. A sluice, as we have seen, consists in principle of a masonry dam pierced with openings, through which the water flows, while the top of the dam is carried up to a level so high that the water does not flow over the top of it. A weir, on the contrary, consists in principle of a low wall across the stream, which flows entirely over the top of the wall, as in a waterfall. The term "regulator" is usually applied to a device of the latter description, and it is employed to regulate the level of the water in a flowing stream. It generally consists of masonry piers fitted with grooves, in which are placed, edge on edge, a number of boards, usually some 6 or 9 inches deep, so that by putting more boards in the grooves the level of the stream is raised, and by removing boards it can be lowered. A weir, or regulator, of this description is always made on the crests of the falls described above, so that the water in the upper reach can be kept at any desired level. The main broad distinction between the sluice and the weir is, then, that the water flows *through* or *under* the sluice, and *over* the weir.

I said just now that "*unfortunately*" Nature had neglected to provide the natural drainage channels with sluices. From a broad point of view, however, it can hardly be called "unfortunate," because the silting-up of these channels forms a part of the process by which the plains of Bengal have been brought into existence. And before proceeding with the details of my subject, I propose to refer briefly to the methods of action of these natural forces, a knowledge of which will go a long way towards helping you to that clear comprehension of the conditions surrounding the problems confronting you, which is so necessary a preliminary to their successful solution in practice.

The plains of Bengal are formed of an alluvial deposit, which has been brought down, in the course of ages, from the great mountain ranges on the north of India, by means of a never-ceasing cycle of operations, which is doubtless familiar to you. First we have the evaporation of water from the surface of the ocean, by the heat of the sun; next, the wind, carrying the moisture-laden air over the land; then the condensation of the vapour by the mountain ranges, causing heavy falls of rain. The water thus precipitated on the mountains finds its way into the valleys, on its course back again towards the sea, and as it goes it not only washes down with it a great deal of the surface-soil, but wears away, slowly but surely, the rocky substance of the mountains themselves, and cuts out for itself channels, gorges, and river beds. One of the most striking

examples in the world of the enormous power of erosion which water possesses is to be seen in the gorge of Niagara in North America, and the same force is constantly at work, in a lesser degree, in every mountain rivulet, stream, and torrent, which helps to swell the volume of the great rivers which issue from the Himalayas, and traverse the plains of Bengal. Thus these great volumes of water, on their way to the sea, are laden with the silt and detritus taken from the mountains; but while the water, after joining the ocean, is again evaporated and condensed in the form of rain, the silt is left behind in the river-beds, on the surface of the country, or on the ocean floor, and thus forms the material of which the plains are built up.

Let us now examine the action of this silting process a little more closely. The silt carried by rivers is of two kinds, the heavier kind consisting of particles of sand, and the lighter kind of soft earthy matter. When a river is in flood, the heavy sandy silt is rolled along the bed, while the soft silt is held in suspension in the water. The heavier particles thus remain in the river-bed, and, accumulating there, gradually raise it, until the banks can no longer contain the floods, which then sweep over the country. If a breach forms in the bank, so that a deep channel is formed, leading away from the river, then the heavy sand may be swept through it on to the fields, destroying their fertility; but when only the top of the flood passes over the bank, without breaching it, the water spreads over the country without violence, carrying with it only the soft silt in suspension, which, as the water subsides, is left on the country, and forms a richly fertile deposit. Thus, in time, the level of the whole country near the river becomes raised. When the bed of the river itself is raised to a certain extent, there is a tendency for breaches to form, and for the river to take a new course altogether, or to form branches, which in their turn take part in the process of fertilising and raising a fresh tract of country. When the course of a river changes before it has completed its work of raising the country near it, we see low-lying tracts of land left unraised; and if the new course of the river is at a considerable distance from its old course, these tracts are again left out from the action of the new river and become permanently low-lying swamps; while the abandoned beds of the old rivers remain in the form of deep *jheels*. A good example of this action is to be found in the many swamps and *jheels* surrounding Jessore, on the Bhairab river. This, once a broad and rapid stream, is now almost stagnant, the bulk of the water having found its way into new channels elsewhere, leaving incomplete its work of raising the country; and leaving also a difficult problem to be solved by the Engineer. The drainage of these tracts sometimes presents great difficulties. During the flood season, the level in the rivers is often higher than in the *jheels*, which thus become filled up with water. When the floods subside, the water is caught, as it were, in a trap, and a channel has to be dug to lead it back again into the river. Even then, the level in the *jheel* may be so low, compared with that in the river, that careful levelling operations may be necessary before it can be pronounced whether drainage is feasible at all; while, even if feasible, the very small difference of level may render it necessary to design the channel so large that the cost of construction may be prohibitively high.

So far we have been following the action of alluvial deposit in inland tracts. Let us now examine the conditions near the coast. We have seen that much of the sand and silt brought down by the river has been expended on the work of raising the inland tracts; but the amount thus left behind is only a small proportion of the whole volume, the bulk of which is carried along into the Bay of Bengal, and deposited on the floor of the sea. From the mouths of the rivers which traverse the Gangetic Delta there extends an enormous heap of sand and mud thus brought down, and "shot," as on to a rubbish heap, into the head of the Bay. For how many miles from the shore this heap extends, or to what height it is piled up on the floor of the Bay, I have not the information to enable me to tell you; but there is no doubt that it exists, and that it is every year extending further southwards and increasing in bulk by millions of tons. The Hooghly alone carries into the sea 39 million cubic yards of mud in one season, a quantity which, if piled up to a height of 30 feet, would cover a square mile of area. This gradual accumulation on the floor of the Bay

takes place out of sight, and out of reach of all observation except the sounding-line of the deep-sea investigator. At the top of the heap, where it approaches the surface, its growth comes within the observation of the Marine Surveyor, on the alert to warn ships of any extension, or change in position, of the shoals at the mouth of the river; while still closer inshore there is a powerful silting action daily at work within sight, resulting in the gradual extension of the land sea-wards.

We may now briefly trace the manner in which this process takes place. You are probably aware that the power of water to retain silt in suspension varies according to its velocity. In still water silt is completely deposited, leaving the water clear, and the higher the velocity the greater is the quantity of silt the water can retain. Consequently when a stream of silt-bearing water has its velocity retarded or checked, silt is at once deposited. Now the water in these tidal estuaries is heavily charged with soft silt, held in suspension, and carried up and down with every ebb and flow of the tide. If deposited at slack water, it is moved again when the tidal currents set in afresh. It is always present in the water, and ready to settle at any place where the current is slack enough to allow it to remain undisturbed. Consider now the case of a small river discharging into this silt-laden estuary. During the dry season, from, say, November to June every year, there is practically no water coming down from the watershed, and the only current is due to the tides, which fill up the channel at every flood, allowing it to empty itself again on the ebb. The river-bed, from its mouth up to the limit of tidal action, forms a reservoir of considerable size, and a large quantity of water runs in and out of it at every tide. Thus at the mouth there must be a fairly rapid current, owing to the passage to and fro of this large volume of water in 12 or 13 hours. As, however, we proceed up the river, the size of the reservoir beyond us becomes smaller, and the current becomes less and less, owing to the smaller volume of water required to fill the smaller reservoir. At a certain point, the velocity will become small enough to allow the silt to remain undisturbed, and it is here that silting tends to begin. As it progresses, the contents of the whole reservoir become smaller, and the silting becomes more rapid and extends towards the mouth, until in time the whole river would silt up; unless the volume of flood-water descending during the rains were sufficient to scour out the bed to its original capacity again. The example I have already quoted, of the Diamond Harbour Creek, is a case where the channels have not been scoured, and have silted up very rapidly.

The same silting action occurs in a very marked degree in the short approach-channels below locks, and the outfall-channels of sluices, and in fact in any short channel with a "dead-end," where there is no reservoir-action, and consequently not enough velocity to keep the silt from depositing.

Another cause of rapid silting is the practice of embanking rivers, to prevent the tidal waters from spilling over the land, which is thus protected from the effect of salt water, and reclaimed. This spill over the banks increases the reservoir-like capacity of the river, and when it is stopped the volume of water entering and leaving with every tide is diminished, the current is slackened, and silting induced.

These causes form a part of the continual process by which the coast-line has been gradually pushed forward into the shallow water at the mouth of the estuary, and the land extended to its present position. It will be seen that, once formed, very little further raising of the land near the coast can occur, and consequently these tracts, for some way inland, are almost at a dead level, and the difficulties of drainage are much increased. In some places the land is actually below the level of high spring tides, and the coast has to be embanked to exclude the tidal water, and prevent the fertility of the land from being destroyed by salt water. Further, high embankments are in many places necessary to protect the land from the destructive effects of the storm-waves caused by cyclones.

We have, then, to deal with a tract of country at practically a dead-level and with no surface-fall, much of it situated below the level of high water, surrounded by high embankments, with silted-up exit-channels, and liable occasionally to extraordinarily heavy falls of rain, the floods from which have to be led away through sluices which, owing to the high range of the tides, can

only discharge during a limited number of hours in the day. To add to the difficulties the sluices have to be constructed at the very mouths of the channels, owing to the liability of the exit-channels to silt up, and in these sites the soil is so treacherous and full of springs that the masonry works are liable to be washed completely away. The situation is completed by the necessity for rigid economy in designing the works, owing to the poverty or want of foresight of the persons for whose benefit they are intended, and the consequent difficulty of raising sufficient funds for their construction. It is difficult to imagine a set of conditions throwing greater impediments in the way of the drainage Engineer. That they are not insuperable is proved by the success (greater or less) of the works that have already been constructed; but I think I have made it clear that great nicety of skill in design is necessary to enable us to obtain the maximum of efficiency combined with the minimum of cost.

In order to attain this object it is necessary to be able to calculate, with a sufficient degree of accuracy, what quantity of water your channels, sluices, and weirs will be able to discharge in a given time, under the conditions which you will meet with in practice. From your text-books and lectures on the subject of Hydraulics you are provided with a quantity of formulæ, easy to understand and to use, giving you the discharges you require. You might think that you have only to put your data into them, and that the rest is a mere matter of arithmetic. It is of course quite necessary first of all to acquire a thorough knowledge of the formulæ of the text-books, and of the methods by which they are derived from first principles, and I shall assume that you already possess this knowledge. These formulæ, however, are only applicable under certain well-defined conditions, and one of the first things that occurs to the young Engineer who uses his brains as he ought to, on attempting to put these results into practice, is that very often these conditions are conspicuous by their absence, or that they are complicated by other conditions for which no allowance is made in the simple formulæ. Under these circumstances, he is driven to an attempt to reconcile the actual conditions with those of his books by assumptions, which may or may not be justified, and which end in "rule-of-thumb" results where there ought to be accurate calculations. As an example, I have seen a case where a sluice had to be designed to discharge into tidal waters at a given rate. The designer had calculated the requisite size by saying "assuming the sluice to discharge under a head of one foot, the velocity will be so much, and the required ventage area so much." Now it is quite safe to say that the sluice would *not* discharge under a head of one foot. Part of the time it would not discharge at all, and part of the time the head would very largely exceed one foot. Results of this sort cannot be looked on as better than mere rule-of-thumb.

The main object of these lectures is to endeavour to remove a few of these difficulties from the path of the young designer, and to attempt to show how, in some cases, the results you have learnt from your text-books may be applied to practical design, under conditions not allowed for in your theory; or how, in other cases, the formulæ must be modified, or re-constructed, to allow for the conditions confronting you in practice. The subject is one which lends itself only too readily to undesirable complications, and I shall endeavour to reduce these complications to as simple a shape as possible, consistently with a sufficient degree of accuracy in the results.

I will first mention briefly some of the conditions which give rise to complications, and indicate generally the means by which I have attempted to arrive at the results, and to express them in a form which will enable you to make practical use of them.

One very common cause of complication is the existence of a velocity of approach. This will be familiar to you, as it is dealt with in the text-books, and you are given formulæ to allow for it. You will see, however, that it is allowed for by introducing into the formula an unknown quantity called the "head due to velocity of approach," and you have to assume successive values for this quantity, and work out your result by trying one value after another until you find one to suit. Now this is a very laborious process, and I think you will admit that it would be a great advantage to get rid of this troublesome unknown quantity altogether, as we do not want it, except as a means to an end. The way in which I have done this will be described

in detail presently. Generally speaking, it consists in expressing the velocity of approach in terms of the area of the channel of approach. To explain this a little more fully; you know that when water is flowing over a weir there is a marked "drop" of level immediately above the weir. I only mention this in order to point out that this is not the "head due to velocity of approach" which I am now speaking of, and that this "drop" can be eliminated in practice by measuring the "head" at some little distance back from the weir. It usually, however, occurs in practice that the water is led up to a sluice or weir along a well-defined and comparatively narrow channel, in which the water flows with a certain definite velocity. It is this velocity which constitutes the velocity of approach we are concerned with. If now we use q to represent the discharge passing through the sluice, and A to denote the area of cross-section of the stream as it approaches the sluice, then the mean velocity of the stream will be $\frac{q}{A}$, and if this quantity be substituted for the "velocity of approach" in the ordinary formula, the troublesome unknown quantity I have referred to can be eliminated. To save you as much labour as possible in making calculations, I have constructed a table giving you the results in every possible case. You have only to calculate what the discharge would be if no velocity of approach existed and then multiply this by the factor given in the table. There is no new principle involved in this. It is merely a labour-saving device which will be found very useful in practice.

A question which may give rise to extreme complications, against which it is as well to guard, is the determination of the "time of discharge." That is, supposing a certain area is covered to a considerable depth with water, which is being drained out through a sluice or over a weir, and it is desired to know how long it will take for the level to fall a certain amount, owing to the discharge. The difficulty in this case arises from the fact that, as the level falls, the "head" over the sluice or weir alters, and thus the rate of discharge is constantly changing. The result has to be deduced from the formula which expresses the rate of discharge at any given moment, by the mathematical process of integrating the *reciprocal* of the formula. Now in some cases this can be easily done, and you are of course familiar with the well-known calculation of the time of emptying (or filling) a lock. But drainage may occur under circumstances calling for the use of one of the more complicated formulæ of discharge, in which case the solution by the rigid mathematical method is extremely complicated or even impracticable. I have got round this difficulty by devising an empirical formula which gives the discharge in the most complicated case, with an error of less than 2 per cent. from the accurate formula, and by using which the time of discharge can be obtained quite easily. But when we come to tidal outfalls, that is, when the sluice or weir discharges from a basin of small area into tidal waters, the level of which rises or falls independently of the fall of level in the drained area, the matter becomes much more complicated. The result I shall give you applies only to one, and that the most simple, of the discharge formulæ. The result is complicated, but it is not easy to see how it can be simplified further, and I have worked out tables to lessen the labour of reducing the result to arithmetic, applicable to every possible case. This case is only applicable to basins of small extent, such as locks or dock basins, where the rate of fall of the basins is considerable compared with the rate of rise and fall of the tide. Fortunately, in the case of most of our large drainage sluices, the rate of fall of level in the drained area is so slow, compared with the rate of rise and fall of the tide, that the former rate can be neglected altogether and the case treated as if the upper surface were stationary. This simplifies matters extremely, and it will be found that most practical cases will be covered by the simple expressions I shall give you showing the true mean discharge of a sluice or weir, during the time that it is being gradually "drowned" by the rising tide, expressed in terms of the initial discharge.

The next cause of complication which I will mention is one of very common occurrence in practice, and I do not think you will find the solution of it in any of the text-books. It is also a very important matter, because it concerns the discharge of water along open channels, both large and small, and has a very considerable effect on the result. The ordinary formula of discharge for an open channel is based on the supposition that the surface of the water is

parallel to the bed of the channel, but we often find in practice that the surface is by no means parallel with the bed, and we are not provided with any accurate means of calculating the discharge. It is usual to use the ordinary formulæ, and reject the bed-slope of the canal in favour of the surface slope; but this is not an exact method. A similar case occurs when the level of a canal is artificially raised by means of a regulator or weir, so that the depth just above the weir is greater than the depth some distance up-stream, and we wish to know what the depth will be at a given distance above the weir. To solve these cases we have to go back to first principles, and form the original equations of fluid motion based on the principles of the conservation of energy. These equations do not, I think, form part of your course, but they are of great interest to the student of higher mathematics, and a thorough understanding of them is necessary to the solution of many difficult problems in Hydraulic Engineering. The differential equation thus obtained is the equation of the curve which the surface of water assumes when it flows with "steady" motion in a uniformly graded channel, the surface not being parallel with the bed; and the integration of this equation supplies us with a knowledge of the difference in depth between any two places at a given distance apart. From the nature of the case it is not to be expected that the result can be a simple one, but I have, as before, reduced as much as possible the labour of calculation by working out tables covering every case likely to arise.

A complicated case of sluice-discharge occurs when the sluice is constructed with a breast-wall in front of the vents. As all our large drainage sluices are built in this way, I propose to go into this case in considerable detail. The exact calculation of the discharge is rather complicated, but fortunately it is possible to arrive at results which in most cases will give a sufficient degree of accuracy by rather more simple methods, which I shall point out to you.

II.—FORMULÆ OF DISCHARGE FOR SLUICES AND WEIRS.

I have now sketched the general nature of the conditions which confront the designer of large drainage works, and some of the causes which give rise to complications in, and variations from, the original formulæ of the text-books. I will next endeavour to show you in detail how these problems may be solved: and I will take first those relating to the capacity of sluices and weirs discharging under varying "heads." We start, of course, with the original formulæ, giving the discharge at any given instant, which you will find in the text-books. They are, as you know, all derived by simple mathematical deduction from the relation, first formulated by Toricelli, and known as the "theoretical velocity of discharge." This relation expresses the fact that, when water issues from a vessel through an orifice at a given depth below the free surface of the water, the velocity with which it issues is equal to that which would be acquired by a heavy particle in falling through the height h. Expressed in the notation we are now using, the relation is written thus

$$v = \sqrt{2gh}.$$

And in practice, as you know, it is necessary to introduce a coefficient to allow for the small defect of the actual velocity from its theoretical value, owing to frictional and other resistances, so that the actual velocity is

$$v = c\sqrt{2gh},$$

c being some proper fraction.

Thus we arrive at the expression for the discharge q through an orifice of area a under a uniform head h, so that every part of the stream issues from the orifice with the velocity "due" to the head h. The discharge is

$$q = a\,v = c\,a\sqrt{2gh}.$$

The discharge over a weir (or notch, as it is called,) of breadth b is arrived at by estimating the sum of the discharges of each elementary strip of the overflow, at a depth x below the free surface, and of height dx. The area of the strip (fig. 3) is bdx, and the velocity of flow is $c\sqrt{2gx}$, so that the discharge of the differential strip is

$$dq = c\sqrt{2gx} \times bdx$$

and the discharge over the whole height h is arrived at by integrating this expression between the limits 0 and h, thus,—

$$q = c\sqrt{2g}\,b\int_0^h x^{\frac{1}{2}}\,dx$$

The result is

$$q = \frac{2}{3}\,c\sqrt{2g}\,b\,h^{\frac{3}{2}}$$

I am afraid that in going into these elementary details I am rather trenching on the province of your regular lecturer on Hydraulics, but it is necessary to dwell somewhat on these original formulæ, in order to familiarize you with the notation used, and to draw your attention to certain points about them. There are in all five original formulæ, expressing the discharges of sluices and weirs, and the conditions to which each is applicable will be explained by the diagrams I shall show you. As regards notation, in every case I have used the letter b to denote the width of the opening; D the total depth of the cross-section of flow; d the depth of the "free" portion of the discharge, *i.e.*, the part above the level of the surface of the tail-water: and h for the greatest "head" of discharge. First of all, I want particularly to point out that the five formulæ may be divided, broadly, into two classes, *viz.*, the "sluice" class and the "weir" class, of which the two simple expressions I have just mentioned are the typical formulæ. The distinction

between the two is that in the "sluice" class the cross sectional area of the whole discharge, *viz.*, bD, is constant, whereas in the "weir" class the area of flow varies with the "head."

The following are the formulæ :—

(*i*) *Completely submerged sluice* (*fig.* 4).

We have just seen that the discharge in this case is

$$q = c\sqrt{2g}\, b\, D \cdot h^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

This is the typical "sluice" formula.

(*ii*) *Freely-discharging sluice* (*fig.* 5).

In deriving this formula from the Torricellian relation, the discharge is treated as the difference of two weir discharges, viz., the discharge over a weir with head h, *less* that with head $(h - D)$. The formula is

$$q = \frac{2}{3} c\sqrt{2g}\, b\, \{h^{\frac{3}{2}} - (h - D)^{\frac{3}{2}}\} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

As, however, the area of discharge bD does not alter with the "head", it comes under the class of "sluice" discharges, according to the definition I have just given you. I will show you that this definition is rightly applicable to this case, as follows :—

Suppose, instead of measuring the head h to the *bottom* of the vent, we measure it to the *centre* of the vent; then, writing h' for this head, and writing d' instead of $\frac{d}{2}$ $\left(\text{or } \frac{D}{2}\right)$ we have

$h = h' + d'$, and $(h - D) = h' - d'$; and the formula becomes

$$q = \frac{2}{3} c\sqrt{2g}\, b\, \{(h' + d')^{\frac{3}{2}} - (h' - d')^{\frac{3}{2}}\}$$

Now, if we expand the two binomials in the large bracket, and subtract (I will leave you to verify the result for yourselves) the formula becomes

$$q = \left(1 - \frac{1}{24}\frac{d'^2}{h'^2} - \frac{1}{128}\frac{d'^4}{h'^4} - \quad \ldots \quad \ldots \right) c\sqrt{2g}\, b\, (2d')\, h'^{\frac{1}{2}}$$

Writing this in the form

$$q = F.\, c\sqrt{2g}\, b\, (2\, d')\, h'^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

you will see that this really corresponds with the typical "sluice" formula, with a factor F introduced, and you will see also that this factor is in most cases very nearly equal to unity. The value of F may also be written in the form

$$F = \frac{\frac{1}{3}\left\{\left(1 + \frac{d'}{h'}\right)^{\frac{3}{2}} - \left(1 - \frac{d'}{h'}\right)^{\frac{3}{2}}\right\}}{\frac{d'}{h'}} \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

a form which is more suitable for calculating the arithmetical values than that given above. You will see that F is most nearly equal to unity when the depth of the vent D is very small compared with the head; and it is *least* equal to unity when $\frac{d}{h'}$ is as large as possible, *i.e.*, when it is equal to unity, and the case becomes one of simple discharge over a weir. I have drawn up this Table I showing the values of this factor F for different values of the ratio $\frac{d'}{h'}$ from which you will see how close the approximation is.

TABLE I.

$\frac{d'}{h'} =$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
$F =$	1·000	·999	·998	·996	·993	·989	·984	·977	·969	·959	·943

For practical purposes, if great accuracy is not essential, it is usual to employ the form

$$q = c\sqrt{2g}\, b\, D\, h^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

as giving a sufficiently close approximation to the true result.

(*iii*) *Partially submerged sluice* (*fig.* 6).

When the surface of the tail-water, below the sluice, is above the level of the floor of the vent, the discharge takes the form of the *sum* of the two preceding cases. That portion of the vent below the level of the tail-water discharges as a submerged sluice, while the portion above the tail-water is in the condition of a freely-discharging sluice.

The formula is written as follows:—

$$q = c\sqrt{2g}\, b \left[\tfrac{2}{3}\{h^{\frac{3}{2}} - (h-d)^{\frac{3}{2}}\} + (D-d)\, h^{\frac{1}{2}}\right] \quad \ldots \quad (6)$$

This, as will be seen presently, is the "general" form of all the other formulæ.

We now come to the "weir" cases, of which there are two, and first we have the typical weir formula, *viz.*—

(*iv*) *Freely discharging weir* (*fig.* 7).

$$q = \tfrac{2}{3} c\sqrt{2g}\, b h^{\frac{3}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

(*v*) *Drowned weir* (*fig.* 8).

When the surface of the tail-water is at a higher level than the rest of the weir, the discharge becomes what is called "drowned," and the formula is a composite one, the upper part discharging as in a "free" weir, while the "drowned" portion discharges as a submerged sluice. The formula is

$$q = c\sqrt{2g}\, b \left\{ \tfrac{2}{3}\, h^{\frac{3}{2}} + (D-d)\, h^{\frac{1}{2}} \right\} \quad \ldots \quad \ldots \quad (8)$$

This may also be written in the forms

$$q = \left(1 - \tfrac{1}{3}\tfrac{h}{D}\right) c\sqrt{2g}\, b\, D h^{\frac{1}{2}} \ldots \quad \ldots \quad \ldots \quad (9)$$

$$\text{and } q = \left\{\tfrac{3}{2}\left(1 - \tfrac{1}{3}\tfrac{h}{D}\right)\left(\tfrac{h}{D}\right)^{\frac{1}{2}}\right\} \tfrac{2}{3}\, c\sqrt{2g}\, b\, D^{\frac{3}{2}} \quad \ldots \quad (10)$$

This completes the five original formulæ. The next thing I wish to draw your attention to is that the complex expression for a partially-submerged sluice may be considered as the "general" formula, and all the other four may be derived from it, by attributing suitable values to the ratios $\frac{d}{h}$ and $\frac{d}{D}$.

Looking at the formula for a partially-submerged sluice, you will see that it may be written in the form

$$q = F\, c\sqrt{2g}\, b\, D h^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

Where F is a factor whose value is

$$F = 1 - \frac{d}{D}\left\{ \frac{\frac{2}{3}\left(1 - \frac{d}{h}\right)^{\frac{3}{2}} - \left(\frac{2}{3} - \frac{d}{h}\right)}{\frac{d}{h}} \right\} \quad \ldots \quad \ldots \quad (12)$$

I leave you to verify this for yourselves. From the figures it is clear that the ratios $\frac{d}{h}$ and $\frac{d}{D}$ must, in every possible case, lie somewhere between the values 0 and 1. I have calculated the value of F for eleven different values of each of these ratios, and the result will be found in the annexed Table II. For any values intermediate between those given, proportional parts may be used, so that the value of F can be found in every possible case. The values of the ratios suitable to each case are as follows:—

(*i*) *Submerged sluice.*—In this case there is no "free" portion of the discharge, and consequently $\frac{d}{h} = 0$ and $\frac{d}{D} = 0$. From the Table it is seen that $F = 1$.

(*ii*) *Freely-discharging sluice.*—Here the discharge is all "free," so that $\frac{d}{D}$ is always $= 1$, while $\frac{d}{h}$ $\left(i.e., \frac{D}{h}\right)$ may have any value between 0 and 1. The corresponding values of F are given in the last column on the right of the Table.

(*iii*) *Partially submerged sluice.*—To this case the whole of the Table is applicable.

(*iv*) *Freely-discharging weir.*—Here the discharge is all "free," and the head h is equal to the depth of discharge D; that is, $\frac{d}{h} = 1$ and $\frac{d}{D} = 1$. It is seen from the table that in this case $F = \frac{2}{3}$.

There is, however, no advantage in using the "general" formula (11) here, because, D being equal to h, and F being, as we have seen, $\frac{2}{3}$, the formula becomes identical with the original formula (7).

(*v*) *Drowned weir.*—Since the depth of "free" discharge is equal to the head, we have here $\frac{d}{h} = 1$; while $\frac{d}{D}$ is equivalent to $\frac{h}{D}$ and may have any value from 0 to 1. The corresponding values of F will be found in the bottom line of Table II.

If we look at the alternative forms of the formula in this case, viz., (9) and (10), we shall see that (9) is identical with the formula we are now considering, because the factor $\left(1-\frac{1}{3}\frac{h}{D}\right)$ or $\left(1-\frac{1}{3}\frac{d}{D}\right)$ is the form which the expression (12) takes when $\frac{d}{h} = 1$. The remaining form (10) is of the "weir" form, instead of the "sluice" form and may be written $q = f\,\frac{2}{3}\,c\sqrt{2g}\,b\,D^{\frac{3}{2}}$. It may be instructive to see what values the factor f assumes for various values of the ratio $\frac{h}{D}$. They are shown in Table III, and we learn from this that the discharge over a weir is only very slightly reduced by "drowning" up to about half the depth of discharge.

These formulæ, and the tables I have given you, will enable you to easily calculate the rate of discharge in any given case when the head remains constant. But when we come to draw mathematical deductions from the original formulæ (as for instance in computing the time of discharge), the simple expression (11) cannot always be used, on account of the variation in the factor F. The general formula (for the partially submerged sluice) is so complicated that no practical result can be obtained from it, and we are obliged to search about for some empirical formula which, while giving results closely agreeing with those given by the exact formula, will also afford us the necessary simplicity of form. I have already pointed out to you one approximate formula, which applies to the case of a freely-discharging sluice, and is obtained by measuring the "head" to the centre of the vent. Expressed in our general notation the formula is

$$q=c\sqrt{2g}\;bD\left(h-\frac{1}{2}D\right)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (13)$$

This is quite accurate enough for our purpose in many cases, though it is rather too wide of the truth when d' exceeds half the head h', that is, when the head, measured to the centre of the vent, is equal to or less than the depth of the vent. It has, further, the fatal defect of being inapplicable to the case of a partially-submerged sluice. I shall now give you an empirical formula which is very close to the truth in *all* cases, and which is applicable to all conditions of discharge. The formula is

$$q=c\sqrt{2g}\;bD\left(h-\frac{5}{9}\frac{d^2}{D}\right)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (14)$$

and it can be expressed in the form

$$q=F.\,c\sqrt{2g}\;bDh^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

where F has the value

$$F=\left(1-\frac{5}{9}\frac{d}{D}\frac{d}{h}\right)^{\frac{1}{2}} \qquad \dots \qquad \dots \qquad \dots \quad (16)$$

In order to compare this formula with the accurate one (11) I have calculated the value of F in (16) for various values of the ratios $\frac{d}{D}$ and $\frac{d}{h}$, and the results are given in Table IV. Now, by comparing Table IV with Table II we can see what are the limits of error of the approximate formula (15) in every possible case. The maximum discrepancy will, I think, be found in the case when $\frac{d}{h}=1$ and $\frac{d}{D}=\cdot6$. The error is, even then, only 2 per cent. of the truth, and this occurs in a "weir" case, when the approximate formula would not, as a rule, be employed. For the "sluice" cases I think the error will hardly ever exceed 1 per cent., and in most cases will be far less than that.

TABLE II.

The figures in the body of the table are values of F, where

$$F=1-\frac{d}{D}\left\{\frac{\frac{2}{3}\left(1-\frac{d}{h}\right)^{\frac{3}{2}}-\left(\frac{2}{3}-\frac{d}{h}\right)}{\frac{d}{h}}\right\}$$

$d/D=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
$d/h=0$	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000
·1	1·000	·997	·995	·992	·990	·987	·985	·982	·980	·977	·975
·2	1·000	·995	·990	·984	·979	·974	·969	·964	·959	·953	·948
·3	1·000	·992	·984	·976	·968	·960	·952	·945	·937	·929	·921
·4	1·000	·989	·978	·968	·957	·946	·935	·924	·914	·903	·892
·5	1·000	·986	·972	·959	·945	·931	·917	·903	·890	·876	·862
·6	1·000	·983	·966	·949	·932	·915	·898	·881	·864	·847	·830
·7	1·000	·980	·959	·939	·918	·898	·878	·857	·837	·816	·796
·8	1·000	·976	·952	·928	·904	·879	·855	·831	·807	·783	·759
·9	1·000	·972	·943	·915	·887	·859	·830	·802	·774	·746	·717
1·0	1·000	·967	·933	·900	·867	·833	·800	·767	·733	·700	·667

TABLE III.

The figures in the body of the table are values of f, where

$$f=\frac{3}{2}\left(1-\frac{1}{3}\frac{h}{D}\right)\left(\frac{h}{D}\right)^{\frac{1}{2}}$$

$h/D=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
$f=$	0·000	·459	·626	·739	·822	·884	·930	·962	·984	·997	1·000

TABLE IV.

The figures in the body of the table are values of F, where

$$F=\left(1-\frac{5}{9}\frac{d}{D}\frac{d}{h}\right)^{\frac{1}{2}}$$

$d/D=$	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
$d/h=0$	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000
·1	1·000	·997	·994	·992	·989	·986	·983	·980	·978	·975	·972
·2	1·000	·994	·989	·983	·978	·972	·966	·960	·955	·949	·943
·3	1·000	·992	·983	·975	·966	·957	·949	·940	·938	·922	·907
·4	1·000	·989	·978	·966	·955	·943	·938	·919	·907	·894	·882
·5	1·000	·986	·972	·957	·943	·928	·907	·898	·882	·866	·850
·6	1·000	·983	·966	·949	·938	·907	·894	·876	·856	·837	·816
·7	1·000	·980	·960	·940	·919	·898	·876	·853	·830	·806	·782
·8	1·000	·978	·955	·938	·907	·882	·856	·830	·803	·775	·745
·9	1·000	·975	·949	·922	·894	·866	·837	·806	·775	·742	·707
1·0	1·000	·972	·943	·907	·882	·850	·816	·782	·745	·707	·667

So far the formulæ have been quoted in terms of h, the head of discharge, and this is the simplest and most generally useful form. The "head" is, however, the difference of *two* quantities, viz., the respective heights of the head-water and tail-water, and it will sometimes, when dealing with tidal outfalls, be found convenient to express the discharge in terms of the height of the head-water only, the effect of the variation of "head" on the discharge being allowed for by a variable factor, calculated for all possible values and tabulated for reference. Using x (see fig. 6) to represent the height of the head-water surface above the floor of the sluice, or crest of the weir, and y the height of the tail-water above the same datum, the most general case of discharge is given by the formula

$$q = F c \sqrt{2g}\, b\, D\, x^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots$$

where the factor F has the value

$$F = \frac{2}{3}\,\frac{\left(1+\frac{1}{2}\frac{y}{x}\right)\left(1-\frac{y}{x}\right)^{\frac{1}{2}} - \left(1-\frac{D}{x}\right)^{\frac{3}{2}}}{\frac{D}{x}} \quad \ldots \quad \ldots \qquad 17$$

The values of F for every possible value of the ratios $\frac{y}{x}$ and $\frac{D}{x}$ are shown in Table V below:—

Table V.

Note.—$\frac{y}{x}$ cannot exceed $\frac{D}{x}$. If y is actually greater than D, then the quantity $(y-D)$ must be subtracted from the actual values of both x and y before reference to the table.

$D/x =$	0	·01	·1	·2	·3	·4	·5	·6	·7	·8	·9	·95	·99	1·0
$\frac{y}{x}$ 0	1·0000	·9973	·9746	·9482	·9207	·8921	·8619	·8300	·7958	·7588	·7173	·6939	...	·6667
·01	...	·9950	...	...	...	...	...	...	...	...	...	...	...	
·1	...	...	·9487	·9353	·9121	·8856	·8567	·8257	·7922	·7550	·7145	·6889	...	·6641
·2	...	...	...	·8944	·8849	·8652	·8405	·8121	·7805	·7454	·7054	·6826	...	·6559
·3	...	...	...	...	·8367	·8290	·8115	·7880	·7599	·7273	·6893	·6674	...	·6414
·4	...	...	...	...	...	·7740	·7679	·7517	·7287	·7000	·6651	·6444	...	·6197
·5	...	...	...	...	...	...	·7071	·7010	·6853	·6621	·6313	·6124	...	·5893
·6	...	...	...	...	...	...	...	·6325	·6265	·6106	·5856	·5691	...	·5481
·7	...	...	...	...	...	...	...	...	·5477	·5417	·5243	·5111	...	·4929
·8	...	...	...	...	...	...	...	...	...	·4472	·4404	·4316	...	·4174
·9	...	...	...	...	...	...	...	...	...	...	·3162	·3139	...	·3057
·95	...	...	...	...	...	...	...	...	...	...	...	·2236	...	·2199
·99	...	...	...	...	...	...	...	...	...	...	...	...	·1000	·0997
1·0	...	...	...	...	...	...	...	...	...	...	...	...	...	·0000

III(a).—TIME OF DISCHARGE, WITH OUTFALL AT CONSTANT LEVEL

The results we have just obtained place us in a position to take up the subject of the Time of Discharge, into an outfall of unvarying level. When water is being discharged through a sluice or weir from a basin of a given extent into an outfall of unlimited area, the surface of the outfall will not be affected by the quantity of water thrown into it, while the surface of the basin will be lowered owing to the amount of water drawn off from it. The rate of lowering of the surface will depend on the area of the basin and the discharge of the sluice or weir. In fig. (9), let A be the area of the basin; h the elevation of the basin surface above the outfall surface after time t; q the rate of discharge of the sluice or weir; T the time taken by the basin surface in falling from the height h_0 to the height h_1; t the time taken in falling from h_0 to h. Then the rate at which the surface is falling, after time t, is $-\frac{dh}{dt}$; the quantity discharged by the sluice in the time dt is $q\,dt$; and the quantity drawn off from the basin, owing to the lowering of surface dh, is $A\,dh$. We thus have the differential equation

$$q\,dt = -A\,dh$$

which can be written

$$dt = -A\frac{dh}{q} \quad \ldots \quad \ldots \quad (18)$$

and the time T is obtained by integrating this equation between the limits O and T on the left side and h_0 and h_1, on the right side, the value of q in terms of h being given by one of the five original formulæ, whichever is applicable. If now we substitute for q its value in the general formula (6) we have an expression which is, to say the least, extremely difficult to deal with, and whose result, if obtainable, would be very complicated. You will now appreciate better the utility of the approximate formula (14).

We will now consider each case in detail.

The simplest case is the first one, that of a submerged sluice. Here, substituting the value of q from equation (1) in equation (18), the differential equation is

$$dt = -\frac{A}{c\sqrt{2g}\,bD}\frac{dh}{h^{\frac{1}{2}}}$$

and the integration, between limits, is

$$T = \frac{2A}{c\sqrt{2g}\,bD}\left(h_0^{\frac{1}{2}} - h_1^{\frac{1}{2}}\right) \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

It is often convenient, in solving questions of this nature, to determine the true *mean* rate of discharge throughout the time. If q_m denotes the true mean discharge, then q_m is equal to the total quantity discharged divided by the total time; that is

$$q_m = \frac{A(h_0 - h_1)}{T} = \frac{(h_0 - h_1)\,c\,\sqrt{2g}\,bD}{2\left(h_0^{\frac{1}{2}} - h_1^{\frac{1}{2}}\right)} = c\,\sqrt{2g}\,bD\frac{h_0^{\frac{1}{2}} + h_1^{\frac{1}{2}}}{2} \qquad (20)$$

That is, the true mean discharge in this case is the arithmetic mean of the initial and final discharges.

Another convenient way of expressing the result is to determine the ratio of the true mean discharge q_m to the initial discharge q_0, which is seen to be

$$\frac{q_m}{q_0} = \frac{1}{2}\left(1 + \frac{h_1^{\frac{1}{2}}}{h_0^{\frac{1}{2}}}\right) \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

We now have $q_m = \left(\frac{q_m}{q_0}\right) q_0$, the values of the factor $\frac{q_m}{q_0}$ for various values of the ratio $\frac{h_1}{h_0}$ being given in the following Table VI.

TABLE VI.

$\frac{h_1}{h_o}=$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
$\frac{q_m}{q_o}=$	·50000	·65811	·72361	·77386	·81623	·85355	·88730	·91833	·94722	·97434	1 00000

The exact formulæ in the cases of the freely-discharging sluice and the partially-submerged sluice are too complicated for practical use in this connection, and we must have recourse to the approximate formula (14); that is, we must substitute the variable $\left(h - \frac{5}{9}\frac{d^2}{D}\right)$ for h, the result being clearly as follows:—

$$T = \frac{2A}{c\sqrt{2g}\,bD}\left\{\left(h_o - \frac{5}{9}\frac{d^2}{D}\right)^{\frac{1}{2}} - \left(h_1 - \frac{5}{9}\frac{d^2}{D}\right)^{\frac{1}{2}}\right\} \quad \ldots \quad \ldots \quad (22)$$

The true mean discharge is, as before, the arithmetic mean of the initial and final discharges; and the ratio of the mean to the initial discharge is

$$\frac{q_m}{q_o} = \frac{1}{2}\left\{1 + \frac{\left(h_1 - \frac{5}{9}\frac{d^2}{D}\right)^{\frac{1}{2}}}{\left(h_o - \frac{5}{9}\frac{d^2}{D}\right)^{\frac{1}{2}}}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

Table VI applies to this also if for the index we write

$$\frac{h_1 - \frac{5}{9}\frac{d^2}{D}}{h_o - \frac{5}{9}\frac{d^2}{D}} \text{ instead of } \frac{h_1}{h_o}.$$

We might also use the exact formula (11), giving the result

$$T = \frac{2A}{Fc\sqrt{2g}\,bD}\left(h_o^{\frac{1}{2}} - h_1^{\frac{1}{2}}\right) \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

but this would only be suitable when the value of F differed very slightly for the initial and final discharges. If this condition is fulfilled, Table VI is applicable as it stands.

We next come to the "weir" cases, and in the simple case of free discharge the differential equation is seen to be

$$dt = -\frac{3A}{2c\sqrt{2g}\,b}\,\frac{dh}{h}$$

the integral of which, between limits, is

$$T = \frac{3A}{c\sqrt{2g}\,b}\left(\frac{1}{h_1^{\frac{1}{2}}} - \frac{1}{h_o^{\frac{1}{2}}}\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

In this case the true mean discharge is *not* equal to the arithmetic mean of the initial and final discharges. The ratio of the true mean discharge to the initial discharge is

$$\frac{q_m}{q_o} = \frac{A(h_o - h_1)}{Tq_o} = \frac{1}{2}\left(1 + \frac{h_1^{\frac{1}{2}}}{h_o^{\frac{1}{2}}}\right)\frac{h_1^{\frac{1}{2}}}{h_o^{\frac{1}{2}}} \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

The values of this ratio are given in the following Table VII:—

TABLE VII.

$\frac{h_1}{h_o}=$	0	·01	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
$\frac{q_m}{q_o}=$	0	·055	·208	·323	·424	·510	·604	·687	·768	·847	·924	1·000

In the last case, that of a drowned weir, the approximate formula (14) is of no use, and the result cannot be reduced to a simple form. Writing δ instead of $(D-d)$, the formula is

$$q = \frac{2}{3} c\sqrt{2g}\, b \left(h^{\frac{3}{2}} + \frac{3}{2}\, \delta\, h^{\frac{1}{2}} \right) \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

It may be noted that D is a variable quantity in this case, but δ is a constant. The differential equation is

$$dt = -\frac{3A}{2c\sqrt{2g}\, b} \; \frac{dh}{h^{\frac{3}{2}} + \frac{3}{2}\, \delta\, h^{\frac{1}{2}}}.$$

The integration, obtained by substituting z^2 for h, is

$$T = \frac{\sqrt{6}A}{c\sqrt{2g}\, b\sqrt{\delta}} \left[\tan^{-1}\left\{\left(\frac{2h_0}{3\delta}\right)^{\frac{1}{2}}\right\} - \tan^{-1}\left\{\left(\frac{h_1}{3\delta}\right)^{\frac{1}{2}}\right\} \right] \quad \ldots \quad (28)$$

The calculation of results is facilitated by Table VIII which gives the value of the factor in brackets for various values of $\frac{\delta}{D_0}$ and $\frac{\delta}{D_1}$.

TABLE VIII.

The figures in the body of the Table are values of the factor

$$\tan^{-1}\left\{\left(\frac{2h_0}{3\delta}\right)^{\frac{1}{2}}\right\} - \tan^{-1}\left\{\left(\frac{2h_1}{3\delta}\right)^{\frac{1}{2}}\right\}$$

$\frac{\delta}{\delta + h_0} =$	...	0	·01	·1	·2	·3	·4	·5	·6	·7	8	·9	1 0
$\frac{2h_0}{3\delta} =$	...	∞	66	6	2·6667	1·5555	1·0	·66667	·44444	·28571	·16667	·07407	0
$\frac{\delta}{\delta + h_1}$	$\frac{2h_1}{3\delta}$	∞											
0	∞	0											
·01	66	·12247	0										
·1	6	·38747	·26500	0									
·2	2·6667	·54049	·12702	·16202	0								
·3	1·5555	·67574	·55327	·28827	·12025	0							
·4	1·0	·78540	·66293	·39793	·23591	·10960	0						
·5	·66667	·85605	76358	·49858	33056	21031	·10065	0					
·6	·44414	·98202	·86015	·59515	·43913	·30683	·19722	·09657	0				
·7	·28571	1·07078	·97531	·69231	·53029	·40404	·29438	·19373	·09716	0			
·8	·16667	1·18305	1·06058	·79558	·63350	·50731	·39765	·29700	·20043	·10327	0		
·9	·07407	1·30493	1·18246	·91740	·75543	·62919	·51053	·41888	·32231	·22515	·12188	0	
1·0	·0	1·57080	1·44833	1·18333	1·02131	·89506	·78540	·68475	·58818	·40102	·38775	·20587	

Formula (10), giving a simpler result, might be used in some cases, where the variation of f is small, the result being

$$T = \frac{3A}{f c\sqrt{2g}\, b} \left(\frac{1}{D_1^{\frac{1}{2}}} - \frac{1}{D_0^{\frac{1}{2}}} \right) \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

The values of f are given in Table III.

III(b).—DISCHARGE INTO A TIDAL OUTFALL, WITH HEAD-WATER AT A CONSTANT LEVEL.

We have now seen how to compute the time of discharge in all cases where the outfall-surface remains at a constant level, while the surface of the head-water falls, owing to the quantity of water drawn off. We next come to a set of conditions which is of very frequent occurrence in Bengal, and is of great importance, because the results will enter into almost every case of drainage into tidal outfalls, and will be continually of use to you in designing such schemes. I allude to the cases where the surface of the head-water falls at so slow a rate, compared with the rise or fall of the outfall surface, that it may be taken as practically at a constant level, while the outfall rises and falls with the tide. In schemes for the drainage of large agricultural areas in Bengal, the rate at which it is usually considered necessary to lower the surface of the drained area is three-quarters of an inch in 24 hours, whereas the rate of rise of the tide may be as much as three feet in one hour, *i.e.*, more than a thousand times as fast. Under these circumstances it is obviously correct to treat the head-level as unvarying.

The final results will, for convenience, be expressed in the form of the ratio $\frac{q_n}{q_o}$, that is, the ratio of the true mean discharge to the maximum discharge; and the values of the factor $\frac{q_n}{q_o}$ will be given in the tables, so that the true mean discharge can be readily found by multiplying q_o by this factor.

First of all, it will be best to consider the manner in which the tide rises. Our knowledge of this is obtained by observation. When we wish to ascertain the tidal *data* at any particular place, we have to fix a gauge, marked in feet and decimals, or feet and inches, in the water at the place, and appoint an observer, provided with a good watch or clock, who observes the height at which the water stands on the gauge at certain definite intervals—say every quarter of an hour, or half an hour. If now we mark off on paper, to a convenient scale, these time-intervals as abscissæ, and the corresponding heights of the gauge as ordinates, we get a series of points through which we can draw a continuous curve, which is called the "tidal curve" Thus, at each point on the curve, the ordinate represents the height of the tidal surface above datum after an interval of time represented by the abscissa. It will be found on the Hooghly and the other tidal rivers on this coast that the shape of the curve is somewhat as shown in the diagram in fig. (10). The front of the curve, *i.e.*, during the rise of the tide, is very nearly a parabola, and so is the portion corresponding with the first part of the ebb. Then the curvature changes, and the remainder of the ebb corresponds fairly nearly with an inverted parabola, but the curvature is somewhat flatter. The duration of the ebb is, roughly, about twice that of the flood.

Now all drainage-sluices in these tidal waters are fitted with flap-shutters outside the vents, opening outwards, so as to allow drainage water to pass from the inside into the tideway, but to close automatically as soon as the tide rises above the level of the water in the drained area. Thus the sluice will only discharge while the tide is below the level of the surface of the water to be drained, and all discharge ceases when the tide rises above that level.

The diagram in fig. (11) represents a sluice, inside which the water is standing at the level AB. CAEBD represents the tide-curve, E being the level of high-water. Suppose the tide is rising, and the sluice is opened when tide-level stands at CD. Then the discharge will continue at a gradually decreasing rate until the tide has risen as high as AB, when the flap-shutter will close, and discharge will cease. The sluice will remain closed until the tide falls again to AB, when the shutter will again open, and allow the discharge to pass, at a gradually increasing rate, until the tide has again fallen to CD, at which level we have supposed the sluice to be closed. Now the *time* during which the sluice remains open is represented on the tide curve by the lengths C M and N D, but the discharge of the sluice during this time (C M + N D) will not be that due to the "head" A M, but something less. Calling the

discharge due to the head A M $= q_o$, the *mean* discharge throughout the time (C M + N D) will be—

$$q_m = \left(\frac{q_m}{q_o}\right) q_o$$

and it is the determination of this factor $\left(\frac{q_m}{q_o}\right)$ with which we are now about to deal.

You will observe that, in the curve C A E B D, the portions C A and B D are very nearly straight lines, indicating that the tide, during the time concerned, rises and falls at a very nearly uniform rate. When, however, high-water is at or below the level of the water which is being drained, the tide-curve would be somewhat as shown by the dotted line F G H, and its parabolic form would have to be taken into account.

Now it is possible that this parabolic form may introduce unpleasant complications into our equations, and the idea suggests itself of "fairing" the tide-curve; that is, of assuming it to be composed of straight lines, as shown in the diagram. As a fact, the tide rises and falls at a varying rate, giving us (see *fig.* 10) the curve A B H C K D L E. Suppose now we *assume* it to rise at a uniform rate, as shown by the straight line A F; then to remain at that height during the time F C M; then to fall at the uniform rates M K and K G; then to remain constant during the time G E. It is obvious that, by placing our straight lines suitably, it will be possible to arrive at the same result as would be given by keeping the curved form. The difficulty is to attribute such values to the times F C; C M; and G E; as will bring about the correct result. The "fairing" of curves in mensuration is a common process, and there we are guided by the areas enclosed between the straight line and the curve; that is, if we were only measuring the areas, we should so adjust the position of the straight line A F as to make the area A B H equal to the area H F C; and similarly we should equalise the areas K D L and L G E. In this case, however, we have to remember that at the top of the tide the "head" is less, and the discharge less, than it is at the bottom of the tide, and that consequently the *lower* areas are of greater relative importance than the *upper* ones. Suppose, to take an illustration, the area H F C was to be covered with pice and the area A B H with rupees, it is clear that, to make them of the same *value*, the former area would have to be much larger than the latter. The difference in the actual case we are considering is not nearly so great as that, but it is evident that, for a true "fairing" of our tide curve, the area H F C may be expected to exceed the area A B H, and the area K D L to exceed the area L G E. We shall see presently that the distance F C varies from about one-third to one-fourth of A N; and we must therefore expect the distance G E to be a good deal less than one-fourth of N E. As N E is about double A N, we should probably be not far wrong in taking G E about equal to F C.

We can now proceed to details, and we will take the simplest case first, *viz.*, that of a completely submerged sluice, with the tide rising at a uniform rate.

In fig. (12), let h_o be the original head; h the head after time t; H the total rise of the tide in the time T; y the rise in time t. The rise being uniform, the tide-curve A B is a straight line, and the value of y in terms of t is $y = \frac{H}{T} t$.

The discharge after time t is

$$q = c\sqrt{2g}\, b\, D\, (h_o - y)^{\frac{1}{2}}$$

and the mean discharge in time T is $q_m = \dfrac{\int_o^T q\, dt}{T}$

Hence $q_m = \dfrac{1}{T} c\sqrt{2g}\, b\, D \displaystyle\int_o^T \left(h_o - \frac{H}{T} t\right)^{\frac{1}{2}} dt$

Substituting z^2 for $\left(h_o - \frac{H}{T} t\right)$, the integration of this is

$$q_m = \frac{c\sqrt{2g}\, b\, D}{H} \cdot \frac{2}{3} \left\{ h_o^{\frac{3}{2}} - (h_o - H)^{\frac{3}{2}} \right\}$$

and since $q_o = c\sqrt{2g}\, b\, D\, h_o^{\frac{1}{2}}$, the value of the ratio we require is

$$\frac{q_m}{q_o} = \frac{2}{3} \frac{h_o^{\frac{3}{2}} - (h_o - H)^{\frac{3}{2}}}{H h_o^{\frac{1}{2}}}$$

or, as it may be written,

$$\frac{q_m}{q_o} = \frac{2}{3}\,\frac{1-\left(1-\frac{H}{h_o}\right)^{\frac{3}{2}}}{\frac{H}{h_o}} \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

We can now calculate the values of $\frac{q_m}{q_o}$ for various values of the ratio $\frac{H}{h_o}$, *i.e.*, the ratio of the maximum rise of tide to the original "head." The results are shown in the following table:—

TABLE IX.

$\frac{H}{h_o}=$	0	·02	·05	·10	·2	·3	·4	·5	·6	·7	·8	·9	·95	1·0
$\frac{q}{q_o}=$	1 00000	·99400	·98760	·97460	·94823	·92075	·89207	·86193	·83002	79588	·75830	·71782	·69302	·66637

We now see that, when $\frac{H}{h_o} = 1$, that is when the high-water level corresponds with the head-water level, so that the original head is just extinguished by the rising tide *at a uniform rate;* then the mean discharge is two-thirds of the maximum, in the case of a submerged sluice.

Let us now see what is the effect of allowing for the parabolic shape of the tide-curve (see *fig.* 13).

The parabolic equation is $\frac{H-y}{H} = \left(\frac{T-t}{T}\right)^2$

and, as in the last case,

$$q = c\sqrt{2g}\; b\, D\,(h_o - y)^{\frac{1}{2}}$$

Thus we obtain the value

$$q_m = \frac{c\sqrt{2g}\,b\,D}{T}\int_0^T \frac{H}{T^2}\left\{T^2\left(\frac{h_o}{H}-1\right)+(T-t)^2\right\}^{\frac{1}{2}} dt.$$

The substitution of z for $(T-t)$ and a^2 for $T^2\left(\frac{h_o}{H}-1\right)$ gives the form

$$q_m = -\frac{\sqrt{2g}\; b\, D\sqrt{H}}{T^2}\int \sqrt{a^2+z^2}\,dz.$$

and the further substitution of $\tan\theta$ for $\frac{z}{a}$ enables the integration to be readily performed, the final result, between limits O and T, being

$$\frac{q_m}{q_o} = \frac{1}{2}\left[1+\frac{1}{2}\,\frac{\frac{h_o}{H}-1}{\left(\frac{h_o}{H}\right)^{\frac{1}{2}}}\log_e\left\{\frac{\left(\frac{h_o}{H}\right)^{\frac{1}{2}}+1}{\left(\frac{h_o}{H}\right)^{\frac{1}{2}}-1}\right\}\right] \quad \ldots \quad \ldots \quad (31)$$

The values of $\frac{q_m}{q_o}$ for various values of $\frac{H}{h_o}$ are shown in Table X below.

TABLE X.

H/h_o =	0	·02	·05	·10	·2	·3	·4	·5	·6	·7	·8	·9	·95	1·0
h_o/H =	∞	50	20	10	5	3·33333	2·50000	2	1 66667	1·4286	1·25	1·1111	1·0526	1·0
q_m/q_o =	1·0000	·99330	·98316	·96595	·93010	·89290	·85346	·81163	·76639	·71689	·66143	·59382	·55554	·50000

Thus the effect of allowing for the parabolic shape, when the head is just completely extinguished, is to reduce the factor from $\frac{2}{3}$rds to $\frac{1}{2}$.

We can now find the correct "fairing" of the tide-curve in the case of a submerged sluice. Suppose the tide to rise at a uniform rate for the time $T-\tau$ and then to remain constant at the full height for the time τ (see *fig.* 14). Then, denoting the final discharge by q_1, where $q_1 = c\sqrt{2g}\ bD\,(h_o - H)^{\frac{1}{2}}$; and calling the factor $\frac{q_m}{q_o}$ in Table IX, F_1, and the factor in Table X, F_2, we have for the whole discharge throughout the time T, according to the "faired" curve,

$$F_1\, q_o\, (T-\tau) + q_1\, \tau$$

and, according to the parabolic curve, the discharge is $F_2\, q_o\, T$.

Now, if the curve is properly faired, these two results must be equal. Hence, equating,

$$(F_1\, q_o - q_1)\, \tau = (F_1 - F_2)\, q_o\, T$$

From this we obtain the relation

$$\frac{\tau}{T} = \frac{F_1 - F_2}{F_1 - \frac{q_1}{q_o}} = \frac{F_1 - F_2}{F_1 - \left(1 - \frac{H}{h_o}\right)^{\frac{1}{2}}} \quad \dots \quad \dots \quad \dots \quad (32)$$

The values of $\frac{\tau}{T}$ for the various values of $\frac{H}{h_o}$ are easily obtained with the help of Tables IX and X, and are shown in Table XI below:—

TABLE XI.

$H/h_o =$	0	·02	·05	·1	·2	·3	·4	·5	·6	·7	·8	·9	·95	·100
$\frac{\tau}{T} =$		·17284	·34366	·33372	·33141	·33018	·32867	·32480	·32208	·31830	·31250	·30203	·29360	25000

The remaining sluice cases and the weir cases can be solved from the general case of a partially-submerged sluice. The solution of this case is as follows:—

Fig. (15) represents a sluice where the initial outfall-level coincides with the floor of the vent. The initial head is h_o, the final head h_1, and the head-water stands at a height Δ above the top of the vent.

We will, as before, take first the case where the tide rises at a uniform rate, so that the rise in time t is $y = \frac{H}{T}\, t$. You will observe, of course, that $H = h_0 - h_1$. Now, after time t, the head is h, and the discharge, as a partially-submerged sluice, is—

$$q = c\,\sqrt{2g}\ b\ \{(h_0 - h)\, h^{\frac{1}{2}} + \tfrac{2}{3}\,(h^{\frac{3}{2}} - \Delta^{\frac{3}{2}})\}$$

Again, since $h = h_0 - y$, it follows that $\frac{dh}{dt} = -\frac{dy}{dt} = -\frac{H}{T}$

Thus we have $q_m = \frac{1}{T}\int_0^T q\ dt = \frac{1}{T}\int_0^T q\,\frac{dt}{dh}\ dh = -\frac{1}{T}\,\frac{T}{H}\int_{h_0}^{h_1} q\ dh.$

Now substituting the value of q, we have

$$q_m = -\frac{c\,\sqrt{2g}\ b}{H}\left\{h_0\int_{h_0}^{h^1} h^{\frac{1}{2}}\ dh - \frac{1}{3}\int_{h_0}^{h^1} h^{\frac{3}{2}}\ dh - \frac{2}{3}\,\Delta^{\frac{3}{2}}\int_{h_0}^{h_1} dh\right\}.$$

The integration of this, in general terms, is

$$q_m = -\frac{2}{3}\,\frac{c\,\sqrt{2g}\ b}{H}\left\{h_0\, h^{\frac{3}{2}} - \frac{1}{5}\, h^{\frac{5}{2}} - \Delta^{\frac{3}{2}}\, h\right\} \quad \dots \quad (33)$$

and between limits

$$q_m = \frac{2}{3}\,\frac{c\,\sqrt{2g}\ b}{H}\left\{h_0\left(h_0^{\frac{3}{2}} - h_1^{\frac{3}{2}}\right) - \frac{1}{5}\left(h_0^{\frac{5}{2}} - h_1^{\frac{5}{2}}\right) - \Delta^{\frac{3}{2}}\left(h_0 - h_1\right)\right\}$$

and since the value of q_o is $\frac{2}{3}\, c\,\sqrt{2g}\, b\, (h_o^{\frac{3}{2}} - \Delta^{\frac{3}{2}})$

we have, dividing q_m by q_o, and then dividing both numerator and denominator by $h_0^{\frac{5}{2}}$

$$\frac{q_m}{q_o} = \frac{\left\{1-\left(\frac{h_1}{h_0}\right)^{\frac{3}{2}}\right\} - \frac{1}{5}\left\{1-\left(\frac{h_1}{h_0}\right)^{\frac{5}{2}}\right\} - \left(\frac{\Delta}{h_0}\right)^{\frac{3}{2}}\left(1-\frac{h_1}{h_0}\right)}{\left(1-\frac{h_1}{h_0}\right)\left\{1-\left(\frac{\Delta}{h_0}\right)^{\frac{3}{2}}\right\}} \qquad \ldots \quad (34)$$

The values of this factor, for different values of the two ratios $\frac{h}{h_0}$ and $\frac{\Delta}{h_0}$, are given in Table XII below :—

TABLE XII.

$h_1/h_0 =$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
$H/h_0 =$	0	·9	·8	·7	·6	·5	·4	·3	·2	·1	0
$\Delta/h_0 = 0$	·80000	·85446	·89368	·92220	·94543	·96360	·97762	·98773	·99420	·99874	1·000
·1	...	·84969	·88916	·92964	·94305	·96211	·97679	·98733	·99459	·99867	1·000
·2	...	...	·88225	·91456	·94007	·96002	·97530	·98753	·99131	·99801	1·000
·3	...	...	...	·90690	·93170	·95644	·97311	·98533	·99378	·99840	1·000
·4	...	...	...	...	92096	·95128	·96991	·98358	·99305	·99823	1·000
·5	...	...	...	...	...	·94367	·96523	·98102	·99197	·99807	1·000
·6	...	...	...	...	...	...	·95799	·97708	·99028	·99749	1·000
·7	...	...	...	...	...	...	...	·97038	·98744	·99678	1·000
·8	...	...	...	...	...	...	...	...	·98170	·99552	1·000
·9	...	...	...	...	...	...	...	...	...	·99118	1·000
1·0	...	...	...	...	...	...	...	...	...	...	1·000

You will see that the case of a weir-discharge occurs when $\Delta = 0$, *i.e.*, the factors applicable will be found in the top line of the table. When in addition the degree of submergence is complete, *i.e.*, when $\frac{h_1}{h_0}$ also $= 0$, or in other words when a weir, discharging freely at first, is gradually submerged by a uniformly-rising tide until the discharge ceases, then the mean discharge during the whole time is $\frac{4}{5}$ths of the initial (maximum) discharge.

We now have to deal with the case (*fig.* 16) where the initial outfall level does not coincide with the floor of the sluice, *i.e.*, when the sluice is "drowned" from the beginning. Suppose the initial depth of submergence to be δ; and we will use h_o to denote the initial head, *not* the head measured down to the floor of the sluice.

The sluice may be treated as made up of two portions, of which the top portion, of depth $(D-\delta)$ acts as a sluice discharging freely to start with, and gradually submerged, just as in the case we have been considering. The lower portion, of depth δ, acts as a completely submerged sluice during the whole time. Now, calling q_{o1} the initial discharge of the upper portion, the mean discharge of this portion throughout the time may be represented by—

$$q_{m1} = f_1 \; q_{o1}$$

Where f_1 is the factor obtained from Table XII, using the index-values $\frac{h_1}{h_o}$ and $\frac{\Delta}{h_o}$.

Similarly the mean discharge of the lower portion may be represented by

$$q_{m2} = f_2 \; q_{o2}$$

Where f_2 is the factor obtained from Table IX, using the index-value $\frac{h_o - h_1}{h_o}$, which is the same as $\frac{H}{h_o}$; and q_{o2} is the initial discharge of the lower portion. Thus the mean discharge of the whole sluice is—

$$q_m = q_{m1} + q_{m2} = f_1 \; q_{o1} + f_2 \; q_{o2}$$

or, since the initial discharge is $q_{o1} + q_{o2}$, we can write

$$\frac{q_m}{q_o} = \frac{f_1 \, q_{o1} + f_2 \, q_{o2}}{q_{o1} + q_{o2}} \qquad \ldots \qquad \ldots \quad (35)$$

When the level of high-water is below the level of the top of the vent, the parabolic shape of the tide-curve has to be allowed for.

We have, as before, $q = c\sqrt{2g}\, b\, \{(h_o - h)\, h^{\frac{1}{2}} + \frac{2}{3}(h^{\frac{3}{2}} - \Delta^{\frac{3}{2}})\}$

$$\text{and } q_m = \frac{1}{T}\int_0^T q\,dt = \frac{1}{T}\int_{h_o}^{h_1} \frac{q}{\frac{dh}{dt}}\,dh$$

$$\text{and } \frac{dh}{dt} = -\frac{dy}{dt}.$$

The value of $\frac{dy}{dt}$ is obtained from the equation of the tide-curve

$$y = T - \frac{H}{T}(T-t)^2$$

$$\text{so that } \frac{dh}{dt} = -\frac{dy}{dt} = -\frac{2H}{T^2}(T-t) = -\frac{2\sqrt{H}}{T}\sqrt{H-y}$$

$$= -\frac{2\sqrt{H}}{T}\sqrt{h-h_1}$$

We now have, for the value of q_m,

$$q_m = -\frac{c\sqrt{2g}\,b}{2\sqrt{H}}\int_{h_o}^{h_1}\left\{ h_o \frac{h^{\frac{1}{2}}\,dh}{(h-h_1)^{\frac{1}{2}}} - \frac{1}{3}\frac{h^{\frac{3}{2}}\,dh}{(h-h_1)^{\frac{1}{2}}} - \frac{2}{3}\Delta^{\frac{3}{2}}\frac{dh}{(h-h_1)^{\frac{1}{2}}}\right\}.$$

The integration is obtained by the substitution $\tan^2\theta = \frac{h-h_1}{h_1}$, and is as follows, in general terms:

$$q_m = -\frac{c\sqrt{2g}\,b}{2\sqrt{H}}\left[\left(h_o - \frac{h_1}{4} - \frac{h}{6}\right)h^{\frac{1}{2}}(h-h_1)^{\frac{1}{2}} - \frac{4}{3}\Delta^{\frac{3}{2}}(h-h_1)^{\frac{1}{2}} + \right.$$

$$\left.\frac{1}{2}h_1\left(h_o - \frac{h_1}{4}\right)\log_e\left\{\frac{1+\left(1-\frac{h_1}{h}\right)^{\frac{1}{2}}}{1-\left(1-\frac{h_1}{h}\right)^{\frac{1}{2}}}\right\}\right]_{h_o}^{h_1}$$

Inserting the limits h_o to h_1, and dividing by $q_o = \frac{2}{3}c\sqrt{2g}\,b\left(h_o^{\frac{3}{2}} - \Delta^{\frac{3}{2}}\right)$, we get as the final result—

$$\frac{q_m}{q_o} = \frac{1}{1-\left(\frac{\Delta}{h_o}\right)^{\frac{3}{2}}}\left[\cdot 62500 - \cdot 18750\left(1-\frac{H}{h_o}\right) - \left(\frac{\Delta}{h_o}\right)^{\frac{3}{2}} + \right.$$

$$\left.\frac{\cdot 09375\left(3+\frac{H}{h_o}\right)\left(1-\frac{H}{h_o}\right)}{\left(\frac{H}{h_o}\right)^{\frac{1}{2}}}\log_e\left\{\frac{1+\left(\frac{H}{h_o}\right)^{\frac{1}{2}}}{1-\left(\frac{H}{h_o}\right)^{\frac{1}{2}}}\right\}\right] \quad \ldots \quad (36)$$

This may also be written—

$$\frac{q_m}{q_o} = \frac{1}{1-\left(\frac{\Delta}{h_o}\right)^{\frac{3}{2}}}\left[\cdot 62500 - \cdot 18750\frac{h_1}{h_o} - \left(\frac{\Delta}{h_o}\right)^{\frac{3}{2}} + \frac{\cdot 09375\left(4-\frac{h_1}{h_o}\right)\frac{h_1}{h_o}}{\left(1-\frac{h_1}{h_o}\right)^{\frac{1}{2}}}\right.$$

$$\left.\log_e\left\{\frac{1+\left(1-\frac{h_1}{h_o}\right)^{\frac{1}{2}}}{1-\left(1-\frac{h_1}{h_o}\right)^{\frac{1}{2}}}\right\}\right]$$

The arithmetical results are shown in Table XIII.—

TABLE XIII.

$h_1/h_0 =$	0	·1	·2	·3	·4	·5	·6	·7	·8	9	1·0
$H/h_0 =$	1 0	·9	·8	·7	·6	·5	·4	·3	·2	·1	·0
$\Delta/h_0=0$	·62500	74641	·81749	·86746	·90962	·94021	·96338	·98017	·99150	·99791	1·00000
·1	...	·73813	·81152	·86312	·90665	·93827	·96210	·97051	·99122	·99784	1·00000
·2	...	...	·79956	·85444	·90074	.93437	·95978	·97823	·99067	·99770	1·00000
·3	...	...	...	·84140	·89185	·92850	·95618	·97627	·98983	·99760	1·00000
·4	...	...	...	...	·87903	·92001	·95008	·97346	·98862	·99722	1·00000
·5	...	...	...	...	—	·90757	·94337	·96933	·98687	99678	1·00000
·6	...	...	...	...	...	...	·93158	·96294	·98412	·99610	1·00000
7	...	...	...	...	...	...	...	·95214	·97947	·99495	1·00000
·8	...	...	...	...	...	...	...	...	·97011	·99264	1·00000
·9	...	...	...	...	...	...	...	...	...	·98501	1·00000
1·0	...	..	...	...	...	...	...	...	...	...	1·00000

As before, the top line of the Table when $\frac{\Delta}{h_0} = 0$ applies to weir discharges. It will be seen that when a weir, discharging freely at first, is gradually submerged until the discharge just ceases at high water, the mean discharge is $\frac{5}{8}$ ths of the maximum, so that the effect of allowing for the curvature of the tide-curve is to reduce the mean discharge from $\frac{4}{5}$ths to $\frac{5}{8}$ths of the maximum.

The "fairing" of the tide-curve (see *fig.* 17) is done on the same principle as before, and we have—

$$\frac{\tau}{T} = \frac{F_1 - F_2}{F_1 - \frac{q_1}{q_0}} \qquad \ldots \qquad \ldots \qquad \ldots \quad (37)$$

Where F_1 and F_2 are the factors obtained from Tables XII and XIII, respectively. But the value of $\frac{q_1}{q_0}$ is more complex than before, for the values are—

$$q_0 = \frac{2}{3} c\sqrt{2g}\, b\, (h_0^{\frac{3}{2}} - \Delta^{\frac{3}{2}})$$

$$\text{and } q_1 = c\sqrt{2g}\, b \left\{ (h_0 - h_1) h_1^{\frac{1}{2}} + \frac{2}{3}(h_1^{\frac{3}{2}} - \Delta^{\frac{3}{2}} \right\}$$

or, as it may be written,

$$q_1 = c\sqrt{2g}\, b \left\{ H(h_0 - H)^{\frac{1}{2}} - \frac{2}{3}(h_0 - H)^{\frac{3}{2}} - \frac{2}{3}\Delta^{\frac{3}{2}} \right\}$$

This supplies us with the value

$$\frac{q_1}{q_0} = \frac{\frac{3}{2}\left(1 - \frac{h_1}{h_0}\right)\left(\frac{h_1}{h_0}\right)^{\frac{1}{2}} + \left(\frac{h_1}{h_0}\right)^{\frac{3}{2}} - \left(\frac{\Delta}{h_0}\right)^{\frac{3}{2}}}{1 - \left(\frac{\Delta}{h_0}\right)^{\frac{3}{2}}} = \frac{\frac{1}{2}\left(\frac{h_1}{h_0}\right)^{\frac{1}{2}}\left(3 - \frac{h_1}{h_0}\right) - \left(\frac{\Delta}{h_0}\right)^{\frac{3}{2}}}{1 - \left(\frac{\Delta}{h_0}\right)^{\frac{3}{2}}} \quad (38)$$

The arithmetical values of $\frac{\tau}{T}$ are given in the following Table XIV:—

TABLE XIV.

$h_1/h_o =$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
$H/h_o =$	1·0	·9	·8	·7	·6	·5	·4	·3	·2	·1	0
$\Delta/h_o =$ 0	·21875	·27290	·28207	·29048	·29057	·29306	·29465	·29577	·30192	·31080	
·1	...	·27287	·28205	·29940	·29972	·29328	·29447	·29622	·29867	·30855	
·2	...	...	·28233	·29950	·29960	·29304	·29446	·29558	·30283	·31597	
·3	...	...	...	·29947	·29057	·29291	·29475	·29008	·30222	·30968	
·4	...	...	...	...	·29050	·29307	·49467	·29573	·30259	·29532	
·5	...	...	...	...	...	·29233	·29453	·29579	·30160	·31778	
·6	...	...	...	...	...	...	·29492	·29000	·30106	·29324	
·7	...	...	...	...	...	...	...	·29572	·30218	·29756	
·8	...	...	...	...	...	...	...	...	·30182	·30638	
·9	...	...	...	...	...	...	...	...	...	·34820	
1·0	...	...	...	...	...	...	...	...	...	...	

TABLE XIV(a).

Table showing values of $\frac{q_1}{q_o}$

$h_1/h_o =$	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
$\Delta/h_o =$ 0	0	·45853	·62010	·73942	·82210	·88380	·92983	·96217	·98387	·99612	1·00000
·1	...	·44085	·61389	·73090	·81638	·88010	·92721	·96093	·98333	·99598	1·0
·2	...	...	·58937	·71882	·80173	·87249	·92261	·95815	·98229	·99573	1·0
·3	...	...	...	·68818	·78723	·86105	·91567	·95478	·98071	·99536	1·0
·4	...	...	...	...	·76107	·84458	·90567	·94930	·97841	·99181	1·0
·5	...	...	...	...	...	·82039	·89101	·94150	·97506	·99401	1·0
·6	...	...	...	...	...	...	·86834	·92931	·96986	·99275	1·0
·7	...	...	...	...	...	...	...	·90870	·96106	·99033	1·0
·8	...	...	...	...	...	...	...	...	·94330	·98612	1·0
·9	...	...	...	...	...	...	...	...	...	·97346	1·0
1·0	...	...	...	...	...	...	...	...	...	...	1·0

Suppose now the sluice, at the beginning of discharge, is submerged to the depth δ over the floor (see *fig.* 18). As before we will now use h_o to denote the initial head, *not* the head measured down to the floor of the sluice. The whole depth of the vent may be divided into two portions, the upper part [$(D-\delta)$ in depth] acting as in the case just considered, while the lower part (δ in depth) acts as a submerged sluice. The mean discharge of the upper part is

$$q_m = f_1\, q_{o1}$$

where f_1 is the factor obtained from Table XIII using the index-values $\frac{h_1}{h_o}$ and $\frac{\Delta}{h_o}$; and the value of q_o is $q_{o1} = \frac{2}{3} c\sqrt{2gb}\,(h_o^{\frac{3}{2}} - \Delta^{\frac{3}{2}})$. The mean discharge of the lower part is $q_m = f_2\, q_{o2}$, where f_2 is the factor obtained from Table X, using the index-value $\frac{H}{h_o}$; and $q_{o2} = c\sqrt{2gb}\;\delta\, h_o^{\frac{1}{2}}$. Thus the mean discharge of the whole sluice is

$$q_m = f_1\, q_{o1} + f_2\, q_{o2} \qquad \ldots \qquad \ldots \qquad \ldots \quad (30)$$

with the above values.

IV.—DISCHARGE WHEN LEVELS OF HEAD-WATER AND OUTFALL VARY SIMULTANEOUSLY.

We have now determined the mean discharges in all cases when the head-level falls and the tail-water level is constant, as well as when the head-level is constant and the tail-water level varies. To complete the investigation we have to consider the cases where the levels of both head and tail-water vary simultaneously.

IV(a).—TIME OF DISCHARGE WHEN BOTH BASINS ARE OF RESTRICTED AREA.

We may take first the case where the outfall, as well as the head-basin, are both of restricted area; *i.e.*, when water drains out of one basin into another of about the same size. The case is not of very great importance, as it does not often occur in drainage problems, but it may be mentioned, in order to complete the subject. We will take first the case of a submerged sluice, and use $x_0 = h_0$ (*fig.* 19) to denote the original difference of the levels of the head and tail-water; x the elevation of the head-water above the same datum after a given time t; and y the rise of the tail-water in the same time; $A_h =$ area of head-basin; and $A_t =$ area of tail-basin. The differential equation is clearly

$$q dt = -A_h\, dx = + A_t\, dy$$

and since the total quantity of water which has been emptied from the head-basin in time t is equal to the quantity which has flowed into the tail-basin, we have

$$(x_0 - x)\, A_h = y\, A_t$$

$$\text{Whence } y = \frac{A_h}{A_t}(x_0 - x)$$

$$\text{The discharge is } q = c\sqrt{2g}\, b\, D\, h^{\frac{1}{2}}$$

$$q = c\sqrt{2g}\, b\, D\, (x-y)^{\frac{1}{2}} = c\sqrt{2g}\, b\, D \left\{ \left(1 + \frac{A_h}{A_t}\right) x - \frac{A_h}{A_t} x_0 \right\}^{\frac{1}{2}}$$

$$q = c\sqrt{2g}\, b\, D \left(\frac{A_h + A_t}{A_t}\right)^{\frac{1}{2}} \left\{ x - \frac{A_h}{A_h + A_t} x_0 \right\}^{\frac{1}{2}}$$

From the differential equation we now have

$$dt = -A_h \frac{dx}{q} = -\frac{A_h}{c\sqrt{2g}\, b\, D} \frac{A_t^{\frac{1}{2}}}{(A_h + A_t)^{\frac{1}{2}}} \frac{dx}{\left(x - \frac{A_h}{A_h + A_t} x_0\right)^{\frac{1}{2}}}$$

The integral of this between the limits x_0 and x_1 gives the result

$$T = \frac{2\, A_h}{c\sqrt{2g}\, b\, D} \frac{A_t^{\frac{1}{2}}}{(A_h + A_t)^{\frac{1}{2}}} \left\{ \left(\frac{A_t}{A_h + A_t} x_0\right)^{\frac{1}{2}} - \left(x_1 - \frac{A_h}{A_h + A_t} x_0\right)^{\frac{1}{2}} \right\}$$

$$T = \frac{2A_h}{c\sqrt{2g}\, b\, D} \frac{A_t}{A_h + A_t} \left\{ x_0^{\frac{1}{2}} - \left(\frac{A_h + A_t}{A_t} x_1 - \frac{A_h}{A_t} x_0\right)^{\frac{1}{2}} \right\} \quad \ldots \quad (40)$$

Now x_0 is equal to h_0, and $h_1 = x_1 - y_1 = x_1 - \frac{A_h}{A_t}(x_0 - x_1)$

$$h_1 = \left(1 + \frac{A_h}{A_t}\right) x_1 - \frac{A_h}{A_t} x_0$$

Hence the time of discharge is

$$T = \frac{2A_h}{c\sqrt{2g}\, b\, D} \frac{A_t}{A_h + A_t} \left(h_0^{\frac{1}{2}} - h_1^{\frac{1}{2}}\right) \quad \ldots \quad \ldots \quad (41)$$

This corresponds with the time of discharge in the simple case of equation (19), with the additional factor $\frac{A_t}{A_h + A_t}$.

The levels will be equalised when $x_1 = y_1 = \frac{A_h}{A_t}(x_0 - x_1)$

i.e., when $\left(1 + \frac{A_h}{A_t}\right) x_1 = \frac{A_h}{A_t} x_0$

and the time taken to equalise the levels is thus

$$T = \frac{2A_h}{c\sqrt{2g}\, b\, D} \frac{A_t}{A_h + A_t} x_0^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (42)$$

The remaining kinds of discharge, viz., those through partially submerged sluices and over weirs, are best solved by using equation (11), with the value of F given in Table II, and if necessary working out the result by dividing the total fall into increments, for each of which the variation of F will be small. The result will of course be in the form just obtained, with $F\, c\sqrt{2g}\, b\, D$ written instead of $c\sqrt{2g}\, b\, D$.

IV(b).—DISCHARGE INTO TIDAL OUTFALL WHEN LEVEL OF APPROACH-CHANNEL VARIES WITH THE TIDE.

The results which I have given in Section III(*b*) apply, strictly speaking, when the head-pool is drained directly into a tide-way. In practice, however, it will more often happen that the drainage has to be conducted from the swamp to the sluice through a long approach-channel, or drainage-cut; and as, in a channel with a free surface, water can only flow from a higher to a lower level, it follows that the surface of the channel, while the discharge is passing, must assume a definite fall, or declivity, in the direction of motion, causing a difference of level between the surface of the channel at its head, near the swamp, and at its tail, just above the sluice. Then, as the tide rises and "drowns" the sluice, the discharge at the tail of the channel is reduced, and the water coming down from the head of the channel tends to accumulate near the sluice and raise the level at the tail, reducing the difference of level and causing a flattening of the longitudinal surface-slope and a consequent reduction of the channel-discharge, corresponding with the reduced discharge through the sluice.

The rise of the tide after "drowning" begins would cause an immediate reduction of the sluice discharge if the head-level at the sluice remained at a constant level; but, as we have seen, the head-level at the sluice (*i.e.*, the tail-level of the channel) also rises, causing an increase of the cross-sectional area of discharge along the channel. This increase of area is accompanied by increases in the velocity, in the coefficient, and in the hydraulic mean depth, all of which causes tend to increase the discharge. If the influence of the increased area exceeds that of the flattened slope, the discharge will actually increase as the tide rises up to a certain maximum, after which the influence of the flattened slope will predominate, and the discharge will decrease. The discharges of both channel and sluice must be considered together, to arrive at a correct estimate of the mean rate of discharge throughout the tide.

The normal condition may be taken as that which occurs at low water, continuing, in the case of a weir, so long as the weir discharges "free" *i.e.*, until "drowning" begins. The sluice and channel should as a rule be designed to suit this condition; subject to alteration if further investigation shows it to be necessary.

Unfortunately the expression for the discharge in the channel is so complicated by the variation in the frictional coefficient and in the wetted perimeter and discharge-area, that a purely analytical solution is impracticable. The channel discharge is given by the equation—

$$q^2 = \frac{2g}{\zeta L} \frac{f\omega^3}{p}$$

where $f =$ total surface-fall in channel
$L =$ length of channel
$p =$ wetted perimeter
$\omega =$ cross-sectional area
$\zeta =$ coefficient of friction

These quantities are illustrated in fig. (20).

The above equation may be written

$$q^2 = C^2 \frac{f\omega^3}{Lp}$$

C being Kutter's coefficient.

The general expression for the sluice-discharge is

$$q^2 = F^2 c^2 2g\, b^2 D^2 h.$$

where F is the factor given in Table II;
or, more conveniently,

$$q^2 = F'^2 c^2 2g\, b^2 D^2 x,$$

where F' is the factor given in Table V.

In practice the *data* are the dimensions of the channel and sluice, and the rate of rise of the tide, so that the problem is to eliminate x and obtain q, the discharge of the channel and sluice, in terms of y, the rise of the tide, which is known in terms of t from the tide-curve. As the quantities f, ω, p and h

are all more or less complicated functions of x, and as the co-efficient ζ (or C) also varies considerably with x, the complication of the problem is obviously prohibitive, even in the cases of the simpler formulæ of discharge. The solution can, however, be obtained in any particular case by the method which I will now point out.

First, the discharges of the channel must be calculated for several different depths, remembering that not only the area of waterway, but also the surface slope will vary with the depth. With these depths as abscissæ and the corresponding discharges as ordinates, plot the curve of discharges; and then the discharge at any intermediate depth can be scaled from this curve. For the sluice-discharge the formula (17) should be used, coupled with Table V, and we require it in the form

$$F = \frac{q}{c\sqrt{2g}\, b\, D\, x^{\frac{1}{2}}} \qquad \ldots \qquad \ldots \quad (43)$$

Now, selecting from the curve of channel-discharges any convenient pair of values of x and q, calculate the value of F from the formula just given, and then, referring to Table V, see (using proportional parts if necessary) what value of $\frac{y}{x}$ corresponds with this calculated value of F. This being determined, the value of y is at once known. Calculating a few values of y in this way, we obtain corresponding values of q and y; and, if we are dealing with a case where the rise of tide occurs at a uniform rate, y is proportional to t, and we can plot a curve with values of t (or y) as abscissæ and q as ordinates. The area of this curve, divided by the maximum abscissa T', will give the value of q_m the mean discharge throughout the time T. The initial discharge q_o being known, the value of the factor $\frac{q_m}{q_o}$ is directly obtained.

If the tide-curve is parabolic (or any shape other than rectilinear) we must, after obtaining corresponding values of q and y as above, find the corresponding value of t from the tide-curve, and plot these values of t as abscissæ to the q-ordinates in the final curve of mean discharge.

The method I have just described was employed in the calculations for the Magra Hât Drainage Scheme, which I will quote in one of my subsequent lectures,* and which will serve as a good practical example.

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* See Section IX(d) of these Lectures.

IV(c). DISCHARGE INTO TIDAL OUTFALL FROM A BASIN OF SMALL AREA.

A third case of simultaneous variation in the levels of both head and tail-water occurs when a basin of small area is drained into a tide-way. The case resembles the one I have described in Section III (a) (see *fig.* 9 and equation 18), with the difference that the quantity h is subject to a double variation, being affected by the fall of the surface in the basin, as well as by the rise of the outfall. It is clear that this case only applies to basins of such small area that the ratio of the rate of fall of surface in the basin to the rate of rise of tide is of appreciable magnitude—say $\frac{1}{10}$th or over. This will usually occur only in the case of dock-basins, or of double locks with basins, to accommodate fleets of boats. It will also be seen that four different cases may occur, as the basin may be either emptying or filling, and the tide may be rising or falling.

The following notation must now be introduced:—

$x_0 = h_0 =$ original height of basin-level above outfall surface.
$x\ \ =$ height of basin-level after the lapse of time t.
$r\ \ =$ rate of rise or fall of tide.
$y\ \ = rt =$ height of outfall above its original level after the lapse of time t.
$rT =$ Total rise of tide in time T.
$h\ \ =$ head of discharge after time t.
$\rho\ \ =$ rate of rise or fall of basin-surface.
$l\ \ =$ alteration of level of basin-surface in time t.

These quantities are illustrated in figs. (21) to (29). The rate of rise or fall of the tide r is taken as a constant. If the rate varies, the whole rise or fall must be divided into portions, for each of which r is sensibly constant, and the calculations made separately for each portion.

You will now see that equation (18) must be replaced by the following:—

$$\pm \frac{dx}{dt} = \frac{q}{A} = \rho \qquad \ldots \qquad \ldots \qquad \ldots \quad (44)$$

where the value of q is expressible in terms of h, the head of discharge, the form depending on the nature of the conditions. As the use of the more complicated formulæ would introduce prohibitive complications, I propose, in this investigation, to consider only the case of the submerged sluice, where

$$q = c\sqrt{2g}\, b\, D\, h^{\frac{1}{2}}.$$

It may be possible to apply these results to other cases of discharge by the use of the factor F and Table II, but such cases are not of common occurrence in practice.

Looking now at the figures (21) to (29) you will see that the head h is increased or decreased by the rise or fall of the basin surface and also by the rise or fall of the tide, that is—

$$\frac{dh}{dt} = \pm \rho \pm r \ \ldots \qquad \ldots \qquad \ldots \qquad \ldots \quad (45)$$

We also have, from equation (44) combined with the discharge equation,

$$\rho = K h^{\frac{1}{2}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \quad (46)$$

Where K is an abbreviation for $\dfrac{c\sqrt{2g}\, b\, D}{A}$.

The solution is obtained from equations (45) and (46) as follows. Differentiating (46),

$$\frac{d\rho}{dt} = \frac{K}{2h^{\frac{1}{2}}} \frac{dh}{dt}$$

Now substituting in this the value of $\frac{dh}{dt}$ from (45) and of $h^{\frac{1}{2}}$ from (46), the differential equation takes the form—

$$\frac{d\rho}{dt} = \frac{K^2}{2\rho} \frac{dh}{dt} = \frac{K^2}{2\rho} (\pm \rho \pm r) \qquad \ldots \qquad \ldots \quad (47)$$

which is easily integrable.

We will now consider separately each of the four cases which can arise.

The first occurs when the basin is being emptied, and the tide is rising. This case is illustrated by fig. (21). Then h is *decreased* by ρ and *decreased* by r, so that equation (45) becomes—

$$\frac{dh}{dt} = -\rho - r$$

and equation (47) becomes

$$-\frac{d\rho}{dt} = \frac{K^2}{2}\,\frac{\rho + r}{\rho}$$

The integration is performed as follows:—

$$-\frac{K^2}{2}\int_0^t dt = \int_{\rho_0}^{\rho} d\rho - r\int_{\rho_0+r}^{\rho+r}\frac{d\,(\rho+r)}{\rho+r}$$

Whence,

$$\frac{K^2\,t}{2} = \rho_0 - \rho - r\,\log_e\left(\frac{\rho_0+r}{\rho+r}\right) \quad \ldots \quad \ldots \quad \ldots \quad (48)$$

Taking the integration up to the time when the "head" is completely extinguished, and both surfaces arrive at the same level, then t becomes T and ρ becomes O, and the equation takes the form—

$$\frac{K^2\,T}{2} = \rho_0 - r\log_e\left(1 + \frac{\rho_0}{r}\right)$$

Dropping the suffix from ρ_0, and writing for K^2 its value $\frac{\rho^2}{h}$ from equation (46) (remembering that the quantities ρ and h now represent their *initial* values) the equation becomes finally—

$$\frac{r\,T}{h} = 2\,\frac{\frac{\rho}{r} - \log_e\left(1+\frac{\rho}{r}\right)}{\frac{\rho^2}{r^2}} \quad \ldots \quad \ldots \quad \ldots \quad (49)$$

This equation gives the value of T, the time required to reduce the "head" to zero from its initial value h, in terms of the ratio ρ/r. Values of $\frac{r\,T}{h}$ corresponding with different values of the ratio ρ/r are given in Table (XV).

Now this equation is in a very inconvenient form, necessitating the use of trial methods, but this is unavoidable. It will, however, be as well to point out the way to use it.

Suppose it is required to find how much the level of a basin will be lowered in a given time t, the initial head h being also given. The following notation will now be used, illustrated in fig. (22).

h = initial head.

h_1 = head after time t.

T = time required to equalise the surfaces, starting with the initial head h.

T_1 = time required to equalise the surfaces, starting with the initial head h_1.

l = lowering of basin-level in time t.

L = " " " " T

L_1 = " " " " T_1

The principle to work on is that the time t is the *difference* between the times of equalising the surfaces from the head h and from the head h_1. The following relations are obvious:

$$t = T - T_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (50)$$

$$l = L - L_1 = h - h_1 - r\,(T - T_1) \quad \ldots \quad \ldots \quad (51)$$

The value of ρ is given by equation (46) as $K\,h^{\frac{1}{2}}$, and the values of h and r are part of the *data*.

First, using the value of ρ/r corresponding with the initial head h, find the value of $\frac{rT}{h}$ from Table (XV). This determines T, since r and h are given. Next, taking a *trial* value of h_1, find the corresponding value of T_1,

also from Table (XV), in the same way. If this value of T_1 satisfies equation (50) then the trial value of h_1 is correct. If equation (50) is not satisfied, select another trial value of h_1, and continue until a value is found to satisfy this test-equation (50). These will be the correct values of h_1 and T_1. The required fall (l) in time t is now given by equation (51).

Next, suppose it is required to find the time taken to lower the basin by a given depth l. As before, begin by finding T_1 and then select successive trial values of h_1, and calculate the corresponding values of T_1 by using Table XV; but in this case the test-equation to be used is (51). When corresponding values of h_1 and T_1 have been found to satisfy equation (51), the value of t is given at once by equation (50).

Secondly, we will consider the case where the basin is being emptied and the tide is falling. (See *figs.* 23, 24, 25 and 26.) You will see that h in this case, is *decreased* by ρ and *increased* by r, so that equation (2) becomes—

$$\frac{dh}{dt} = -\rho + r \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

and equation (47) is—

$$\frac{d\rho}{dt} = \frac{K^2}{2}\,\frac{r-\rho}{\rho} \quad \ldots \quad \ldots \quad \ldots \quad (53)$$

In order to determine the limits of integration suitable to the conditions of the present case, I must now call your attention to the fact that, in this case, the rate of increase of both h and ρ depends on the *difference* of the quantities r and ρ. Suppose, now, that both surfaces are originally at the same level, and that, as the tide falls, the value of h, starting at *zero*, begins to increase (*figs.* 23 and 24). The quantity ρ, which was at first *zero*, will increase concurrently with h (see equation 46), until ρ approaches the value of r, which has, you will remember, a constant value. As ρ approaches r, the quantities $\frac{dh}{dt}$ and $\frac{d\rho}{dt}$ approach the value *zero* (see equations 52 and 53); that is, both ρ and h tend to become constant. Thus whenever ρ is less than r, its value will continually increase until it becomes equal to r, after which it will remain constant and equal to r, and the head and discharge will also remain constant. It can be shown mathematically that ρ will *always* remain constant, when once it reaches the value r, as follows:—

By successive differentiation from equation (53), we have—

$$\frac{2}{K^2r}\,\frac{d^2\rho}{dt^2} = -\rho^{-2}\,\frac{d\rho}{dt}$$

$$\frac{2}{K^2r}\,\frac{d^3\rho}{dt^3} = 2\rho^{-3}\,\frac{d\rho}{dt} - \rho^{-2}\,\frac{d^2\rho}{dt^2}$$

$$\frac{2}{K^2r}\,\frac{d^4\rho}{dt^4} = -6\rho^{-4}\,\frac{d\rho}{dt} + 4\rho^{-3}\,\frac{d^2\rho}{dt^2} - \rho^{-2}\,\frac{d^3\rho}{dt^3}$$

and generally

$$\frac{2}{K^2r}\,\frac{d^n\rho}{dt^n} = -\rho^{-2}\,\frac{d^{n-1}\rho}{dt^{n-1}} + N_1\,\rho^{-3}\,\frac{d^{n-2}\rho}{dt^{n-2}} - N_2\,\rho^{-4}\,\frac{d^{n-3}\rho}{dt^{n-3}} + \ldots\ldots$$

$$\ldots\ldots + (-1)^{n-2}\,N_{n-3}\,\rho^{-(n-1)}\,\frac{d^2\rho}{dt^2} + (-1)^{n-1}\,N_{n-2}\,\rho^{-n}\,\frac{d\rho}{dt}$$

where N_1, N_2, &c. are numerical co-efficients.

From this we see at once that when $\rho=r$, *all* the successive differential coefficients of ρ vanish, and therefore the value of ρ at which this occurs is neither a maximum nor a minimum. Thus ρ must remain constant, since $\frac{d\rho}{dt}$ is *zero*. In this case ρ must always be either less than or equal to r.

Suppose, on the other hand, that the sluices are opened when the "head" is so large that the initial value of ρ exceeds that of r (*figs.* 25 and 26). Then the basin-surface falls faster than the tide-way surface, and the head (and consequently ρ) will continually diminish until ρ becomes equal to r, after which the surfaces will continue to fall at the same rate r, and the head and discharge will remain constant. In this case ρ must always be either greater than or equal to r. You can now see that the limits of integration applicable

will depend on whether ρ is less or greater than r. Taking the former case first, the suitable limits will be measured from the time that has elapsed (or that would have elapsed) since the surfaces started level. That is, T will be the time that would be required to bring the surfaces to a difference of level h, assuming them to have started at the same level. Returning to equation (53) and integrating—

$$\frac{K^2}{2}\int_0^T dt = \int_0^\rho \frac{\rho\, d\rho}{r-\rho} = -\int_0^\rho d\rho - r\int_r^{r-\rho} \frac{d(r-\rho)}{r-\rho}$$

Whence

$$-\frac{K^2 r\, T}{2\, r^2} = \frac{\rho}{r} + \log_e\left(1 - \frac{\rho}{r}\right)$$

and, since $K^2 = \frac{\rho^2}{h}$

$$\frac{r\,T}{h} = -2\,\frac{\frac{\rho}{r} + \log_e\left(1 - \frac{\rho}{r}\right)}{\frac{\rho^2}{r^2}} \qquad \ldots \qquad \ldots \qquad (54)$$

The values of $\frac{r\,T}{h}$ corresponding with different values of $\frac{\rho}{r}$ will be found in Table XVI.

The trial method has to be employed, as before. The notation used is illustrated in fig. (24). Suppose it is required to find the time t, in which the basin-surface will fall through a given difference of level l, starting with a given initial head h. From these data calculate the value of ρ/r, which, under the present supposition, is less than 1, and find $\frac{r\,T}{h}$ from Table XVI. This gives the time T in which the surfaces would have arrived at their initial position if they had started level.

Next, take successive trial values of h, and find $\frac{r\,T_1}{h_1}$ from Table XVI, until values are found which satisfy the equation—

$$l = L_1 - L_0 = r(T_1 - T) - (h_1 - h) \qquad \ldots \qquad \ldots \quad (55)$$

The value of t is now given at once by $t = T_1 - T$ ... (56)

In case the value of t is given, and it is required to find l, the test-equation for the trial values of h_1 will be (56), and the required value of l will be found from (55).

We now come to the case where ρ is greater than r (see *figs.* 25 and 26), which, as we have seen, occurs when the sluices are opened under a considerable "head" which at once begins to diminish and continues to do so until ρ tends to become equal to r. As a matter of fact ρ can never become actually equal to r in less than an infinite time (because the term $\log_e\left(1 - \frac{\rho}{r}\right)$ becomes infinite when $\rho = r$), and we must consequently choose as one of the limits of integration some value of ρ slightly greater than r, say $\rho = 1{\cdot}01r$. The integration of (53) will now be as follows:—

$$\frac{K^2}{2}\int_0^T dt = -\frac{\rho\, d\rho}{\rho - r}$$

$$= -\int_\rho^{1\cdot01r} d\rho - r\int_{\rho-r}^{\cdot01r} \frac{d(\rho-r)}{\rho-r}$$

$$\frac{K^2\,T}{2} = \rho - 1{\cdot}01r + \log_e\left(\frac{\rho-r}{\cdot01r}\right)$$

$$\frac{K^2\,T}{2r} = \frac{\rho}{r} - 1{\cdot}01 + \log_e 100 + \log_e\left(\frac{\rho}{r} - 1\right)$$

Substituting the value $K^2 = \frac{\rho^2}{h}$ this becomes—

$$\frac{rT}{h} = 2\,\frac{3{\cdot}5952 + \frac{\rho}{r} + \log_e\left(\frac{\rho}{r} - 1\right)}{\frac{\rho^2}{r^2}} \quad \ldots \quad \ldots \quad (57)$$

The values of $\frac{rT}{h}$ satisfying this equation are shown in Table XVII.

The method of working is illustrated in fig. (26).

As before, obtaining from the data the value of ρ/r, find $\frac{rT}{h}$ from Table XVII. Next, taking successive trial values of h_1, find $\frac{rT_1}{h_1}$ from the same table, until values are obtained which satisfy the test-equation

$$l = L_0 - L_1 = rT + h - \left(\frac{1{\cdot}01r}{K}\right)^2 - \left\{rT_1 + h_1 - \left(\frac{1{\cdot}01r}{K}\right)^2\right\}$$

$$= h - h_1 + r(T - T_1) \quad \ldots \quad \ldots \quad \ldots \quad (58)$$

and, of course,

$$t = T - T_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (59)$$

The third case occurs when the basin is being filled and the tide is rising. In this case h is increased by r and decreased by ρ, so that

$$\frac{dh}{dt} = r - \rho$$

Taking first the case where ρ is less than r (*fig.* 27); then, as before, ρ must always remain less than r, and T will be the time that would have elapsed in bringing the surfaces to a difference of level h, if they had started level. Equation (54) and Table XVI apply, and equations (55) and (56).

When ρ exceeds r, the conditions, illustrated by fig. (28), are similar to the former case, ρ being always greater than r, and T being the time that would elapse in reducing the difference of level from h (or h_1) to the amount $\left(\frac{1{\cdot}01\,r}{K}\right)^2$. Equation (57), with Table XVII, must be used, and equations (58) and (59).

There remains the fourth case, when the basin is being filled on a falling tide (see *fig.* 29). The head is now decreased by ρ and decreased by r, and the case is similar in every way to the first case, when the basin was being emptied on a rising tide. Equation (49) with Table XV must be used, and equations (50) and (51).

Finally it may be mentioned that, when ρ is very nearly equal to r, the solution is very simply obtained from equations (44) and (46), with the condition that $l = \rho t$.

TABLE XV.

Showing corresponding values of $\frac{\rho}{r}$ and $\frac{rT}{h}$, where

$$\frac{rT}{h} = 2\,\frac{\frac{\rho}{r} - \log_e\left(1 + \frac{\rho}{r}\right)}{\frac{\rho^2}{r^2}}$$

ρ/r =	..	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	1·1	1·2	1·3
rT/h =	...	·939	·884	·836	·788	·756	·722	·691	·663	·637	·614	·592	·572	·553
ρ/r =	...	1·4	1·5	1·6	1·8	2·0	2·2	2·4	2·6	2·8	3·0	3·25	3·5	3·75
r/Th =	...	·535	·519	·505	·476	·451	·428	·408	·390	·374	·359	·341	·326	·312
ρ/r =	...	4·0	4·5	5·0	6·0	7·0	8·0	9·0	10	12	15	20	50	100
rT/h =	...	·299	·276	·257	·225	·201	·181	·165	·152	·130	·109	·085	·037	·019

TABLE XVI.

Showing corresponding values of $\frac{\rho}{r}$ and $\frac{rT}{h}$, where

$$\frac{rT}{h} = -2\,\frac{\frac{\rho}{r} + \log_e\left(1 - \frac{\rho}{r}\right)}{\frac{\rho^2}{r^2}}$$

ρ/r = ...	·1	·2	·3	·4	·5	·55	·6	65	·7	·75	·8	·85	·0	·93	·95	·97	·98	·99	1·00
rT/h = ...	1·074	1·158	1·269	1·385	1·545	1·649	1·757	1·898	2·056	2·262	2·530	2·800	3·574	3·999	4·584	5·392	6·106	7·377	∞

TABLE XVII.

Showing corresponding values of $\frac{\rho}{r}$ and $\frac{rT}{h}$, where

$$\frac{rT}{h} = 2\,\frac{3{\cdot}5952 + \frac{\rho}{r} + \log_e\left(\frac{\rho}{r} - 1\right)}{\frac{\rho^2}{r^2}}$$

ρ/r = ...	1·010	1·011	1·012	1·013	1·014	1·015	1·010	1·018	1·020	1·025	1·030	1·04	1·06	1·08	1·10	1·12	1·15
rT/h = ...	0·000	0·188	0·303	0·517	0·662	0·707	0·922	1·160	1·362	1·778	2·109	2·619	3·278	3·686	3·955	4·187	4·307
ρ/r = ...	1·18	1·10	1·20	1·25	1·30	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·2	2·4	2·6	2·8	3·0
rT/h = ...	4·390	4·413	4·125	4·127	4·308	4·102	3·013	3·600	3·418	3·103	2·080	2·798	2·470	2·199	20·12	1·764	1·620
ρ/r = ...	3·25	3·50	3·75	4·0	4·5	5·0	5·5	6·0	7	8	9	10	12	15	20	50	100
rT/h = ...	1·450	1·307	1·180	1·037	0·825	·709	·701	·622	·500	·423	·362	·316	·250	·169	·133	·016	·0214

V.—VELOCITY OF APPROACH.

It will now be appropriate to deal with the very important subject of "Velocity of Approach," which I have already mentioned. We have seen that the velocity of flow through a submerged sluice is $c\sqrt{2gh}$, h being the head and c a coefficient, allowing for the effects of friction and contraction. If, now, the area of sluice-opening is a, the discharge is given by the equation $q = c\,a\sqrt{2gh}$. Thus the theoretical velocity of discharge is $v = \sqrt{2gh} = \frac{q}{c\,a}$, so that $c\,a$ may be looked on as the "virtual" area of waterway. In this we have taken the sluice as discharging direct from a basin of large extent, so that the water in the basin has no general velocity of any appreciable amount, in the direction of the sluice. Practically, however, drainage water is generally led towards a sluice through a channel of restricted cross-sectional area, so that the water approaches the sluice with a velocity $\frac{q}{\Omega}$ where Ω is the cross-sectional area of the channel. In the first case, the "head" necessary to impart the velocity $\frac{q}{c\,a}$ to the water was $h = \frac{q^2}{2g\,(c\,a)^2}$. In the second case, the water already possesses the velocity $\frac{q}{\Omega}$, corresponding with the "head" $\frac{q^2}{2g\,\Omega^2}$, so that the balance of head required to make up the velocity is only

$$h = \frac{q^2}{2g\,(c\,a)^2} - \frac{q^2}{2g\,\Omega^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \; (60)$$

Bernouilli's theorem supplies us with an accurate method of determining the relation between the head and the discharge, and on account of the importance of the subject, I propose to deal with it in detail.

In figure (30), ABC represents a filament, or stream-line, along which water is flowing from the approach channel, through the sluice, away down the recess-channel, or outfall-channel; and to make the demonstration complete we will suppose that both approach and recess-channels are of restricted cross section, so that the water approaches the sluice with the velocity u_1, and flows away from it also with a definite velocity u_3. The velocity through the sluice itself is u_2, and z_1, z_2, z_3, are the elevations of the stream-line at the three places we are considering. Then the total head at A is as follows:—

Head due to elevation	...	... $= z_1$
„ pressure	...	... $= y_1 - z_1$
„ velocity	...	... $= \frac{u_1^2}{2g}$

The total head at B is as follows:—

Head due to elevation	...	... $= z_2$
„ pressure	...	... $= y_2 - z_2$
„ velocity	...	... $= \frac{u_2^2}{2g}$

and let the proportion of head lost in friction and contraction in the sluice be represented by ... $= \zeta_0 \frac{u_2^2}{2g}$

Equating the total head at A to that at B we have

$$(1 + \zeta_0)\frac{v_2^2}{2g} = y_1 - y_2 + \frac{u_1^2}{2g}$$

That is, $u_2 = \frac{1}{\sqrt{1+\zeta_0}} \sqrt{2g}\left(h + \frac{u_1^2}{2g}\right)^{\frac{1}{2}}$

It is usual to use the coefficient c to denote the quantity $\frac{1}{\sqrt{1+\zeta_0}}$, so that we have, as the formula of discharge when velocity of approach exists,

$$q = a\,u_2 = c\sqrt{2\,g}\,a\left(h + \frac{u_1^2}{2g}\right)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (61)$$

The quantity $\frac{u_1^2}{2g}$ is usually called the "head due to velocity of approach." Next, the total head at C is as follows:—

Head due to elevation	$= z_3$
" pressure	$= y_3 - z_3$
" velocity	$= \frac{u_3^2}{2g}$
Head lost in friction and contraction in the sluice	$= \zeta_0 \frac{u_2^2}{2g}$
Head lost in eddying motion in the tail-channel, owing to the change of velocity from u_2 to u_3	$= \frac{u_2^2}{2g} - \frac{u_3^2}{2g}$

The total head at C is thus $= (1+\zeta_0)\frac{u^2}{2g} + y_2$, which is identically equal to the total head at B; from which we see that the existence of the residual velocity u_3 has no effect on the discharge of the sluice.

If now, in equation (61) we substitute for u_1 its actual value $\frac{q}{\Omega}$, the equation can be written in the form

$$\frac{q^2}{c\;a^2} - \frac{q^2}{\Omega^2} = 2\,g\,h$$

which is identical with the expression (60) I gave you just now. This may also be written in the form

$$\left.\begin{array}{l} q = \dfrac{1}{\left(1 - \dfrac{c^2 a^2}{\Omega^2}\right)^{\frac{1}{2}}}\,c\,a\,(2\,g\,h)^{\frac{1}{2}} \\ \text{That is, } q = C_s\,c\,a\,(2gh)^{\frac{1}{2}} \end{array}\right\} \quad \ldots \quad \ldots \quad (62)$$

Where C_s is a factor allowing for the effect of velocity of approach and having the value $\frac{1}{\left(1 - \frac{c^2 a^2}{\Omega^2}\right)^{\frac{1}{2}}}$. The values of C_s for various values of the ratio $\frac{c\,a}{\Omega}$ are given in Table XVIII.

The factor we have just obtained applies to cases of sluice-discharge. The factor applicable to weir-discharges is not quite so easily arrived at. The fundamental equation, which you will find in the text-books, is—

$$q = \tfrac{2}{3}\,cb\,\sqrt{2g}\,\{(h + \mathfrak{h})^{\frac{3}{2}} - \mathfrak{h}^{\frac{3}{2}}\}$$

Where $\mathfrak{h}$ is the head due to velocity of approach, and is $= \frac{u_1^2}{2g}$.
This equation can be written as follows:—

$$q = C_w\,\tfrac{2}{3}\,cb\,\sqrt{2g}\,\mathfrak{h} \quad \ldots \quad \ldots \quad \ldots \quad (63)$$

Where C_w, the factor allowing for the effect of velocity of approach, is

$$C_w = \left\{\left(1 + \frac{\mathfrak{h}}{h}\right)^{\frac{3}{2}} - \left(\frac{\mathfrak{h}}{h}\right)^{\frac{3}{2}}\right\} \quad \ldots \quad \ldots \quad (64)$$

Now, expressing u_1 in terms of q and Ω, we have

$$q = \Omega\,u_1 = \Omega\,(2g\,\mathfrak{h})^{\frac{1}{2}}$$

Equating this to (63) above, and using a to denote the area of waterway bh, it follows that

$$\Omega (2g\mathfrak{h})^{\frac{1}{2}} = C_w \frac{2}{3} ca\, (2gh)^{\frac{1}{2}}$$

$$\text{Whence } \frac{ca}{\Omega} = \frac{3}{2\,C_w}\left(\frac{\mathfrak{h}}{h}\right)^{\frac{1}{2}} \qquad \dots \qquad \dots \quad (65)$$

The corresponding values of $\frac{ca}{\Omega}$ and C_w, are shown in Table XVIII, which is calculated as follows. Select any convenient values of $\frac{\mathfrak{h}}{h}$, ranging from 0 upwards, and for each value calculate firstly the value of C_w from (64) and secondly the value of $\frac{ca}{\Omega}$ from (65). Tabulate all the values thus obtained; and then any required intermediate values of $\frac{ca}{\Omega}$ (and C_w) can be obtained by using proportional parts, or by plotting the calculated values in curves and scaling off the required intermediate values. The accuracy of the results can be tested by trying whether the results in Table XVIII satisfy both equations (64) and (65). The values of $\frac{\mathfrak{h}}{h}$, not being required except as stepping-stones to the values of $\frac{ca}{\Omega}$ and C_w, are not quoted in the table.

It will be seen that the values of C_w correspond fairly closely with those of C_s. The factor C_s may be used for all the "sluice" cases, *i.e.*, in all cases where the top of the vent is below the surface of the head-water.

TABLE XVIII.

$\frac{ca}{\Omega}$ = ...	0	·05	·10	·15	·20	·25	·30	·35	·40	·45	·50	·55	·60	·65	·70	·75	·80	·85	·90	·95	1·00
C_s = ...	1·000	1·0013	1·005	1·0115	1·021	1·033	1·048	1·068	1·091	1·120	1·155	1·193	1·250	1·316	1·400	1·512	1·667	1·893	2·294	3·203	∞
C_w = ...	1·000	1·0015	1·006	1·0140	1·026	1·040	1·059	1·081	1·108	1·141	1·180	1·228	1·290	1·361	1·453	1·572	1·836	2·013	2·410	3·830	∞

VI.—SLUICE WITH BREAST WALL IN FRONT OF VENTS.

I HAVE already sketched the general nature of the conditions under which large sluices, draining into tidal waters, are constructed in Bengal. The tidal water is excluded from the land by embankments, in which sluices are necessary, to allow of the rain-water being drained off the land at low tide. The land is low, and the range of tide considerable, so that, while the level of the tide-way at high water may be a good many feet above the land, the low-water-level may be a very considerable depth below it. Thus, it may very well happen that the level of low-water is some 12 feet or more below the level of the flood-water lying on the protected lands, and with this "head" of discharge the velocity through the sluice would be about 20 feet per second. Now the sluices are, as I have mentioned, built near the mouths of khals on alluvial soil, which is sometimes treacherous and full of springs. The material used is brick, and it is desirable, for the sake of economy, not to use too great thicknesses of masonry. A sluice of this nature would be subjected to very heavy vibration and strain by such a high velocity, even without the danger of scour in the channel below the sluice. Failure is, however, most liable to occur by a deep pot-hole being scoured out just below the apron of the sluice and this cavity extending backwards underneath the apron, which, being undermined, falls into the hole, until the sluice itself is threatened. Cases have occurred of sluices being completely washed away in this manner. To throughly protect the channel below the sluice against scour would be very expensive, even if practicable; but it is not necessary to do so, if we can adopt means to ensure that the velocity through the sluice shall never exceed some safe limit. This is always done now by the construction of a breast-wall in front of the vents (*i.e.*, upstream of them), so that the water has to flow over this wall, as over a weir, before passing through the vents. Thus supposing the flood level on the country to stand at + 12·00, and the crest of the breast-wall to be at + 8·00, it follows that the velocity can never exceed that due to a weir discharge with a "head" of 4 feet. The discharge would begin when the tide fell below the level 12·00, would gradually increase until the level 8·00 was reached, and after that would increase no more however low the tide fell, instead of increasing to a velocity of perhaps 20 feet per second, as it might were there no breast-wall. Here, then, we have a new case of sluice discharge, where the water passes over a weir situated close in front of the sluice vents, and the determination is somewhat complicated.

The special notation employed in this connection is as follows :—

H = height of surface in head-pool.
B = height of weir wall.
x = height of surface in the intermediate space, between the weir and the vents.
R = height of surface in tidal outfall.
D = height of vents.
b_w = breadth of weir wall.
b_x = ,, of intermediate space.
b_s = ,, of waterway of vents.
c_w = coefficient of discharge over the weir.
c_s = ,, ,, through the vents.
C_s = ,, velocity of approach to the vents.

The discharge occurs first over the weir into the space between the weir and the vents, and secondly out of this space, through the vents, into the tideway. As the tide rises, the vents are gradually "drowned," causing the water in the intermediate space to rise also and "drown" the discharge over the weir. The waterway in this intermediate space may be only slightly greater than that through the vents, and there will, therefore, be a considerable velocity in it, as the discharge passes forward to the vents. As we have seen above, when considering the subject of velocity of approach, this velocity will not

affect the weir discharge, but it will constitute a fairly high velocity of approach to the vents, and thus, in estimating the velocity through the vents, we must employ the approach coefficient C_s. If the waterway in the space were equal to the waterway through the vents, this coefficient would be infinite, implying that the heading up would be zero; that is, that $x = R$. Practically the waterway in the space will be the larger, especially where there is more than one vent, and the waterway of the sluice is reduced by the thickness of the piers between the vents. Even when there is only one vent, there will be some coefficient of contraction of the flow, and the virtual area of the ventage may be taken as $c_s\, b_s\, x$ against the area $b_x\, x$ of the space. In any case the heading up of the level in the intermediate space, caused by the vents, will be less than it would be if the space contained "dead" water. The question next arises as to the point at which the width b_x and the depth x in the intermediate space are to be measured, as the wing-walls gradually converge from the weir to the vents. As the velocity in the space has no effect on the weir discharge, it will make no difference to the weir where the dimensions are reckoned; and it appears correct to regard the sluice discharge as affected by the velocity immediately above it, rather than by the velocity in any more remote place. Hence the dimensions b_x and x will be measured close to the vents. The height x is hardly susceptible of practical measurement, as the water in the space is in a state of violent disturbance, and in some cases is subject to a general oscillation, its surface rising and falling at intervals. The height x must therefore be looked on as the equivalent height due to the mean pressure, reckoned over a considerable time, in the filaments passing through the space, immediately above the vents.

We can now form the equations of discharge for each condition that presents itself during the rise of the tide. First, we will take the simplest case, which occurs at first, when the level of the tide is low, and the level in the intermediate space is below the crest of the weir-wall (see *fig.* 31). So far as the weir is concerned, we have here a simple case of "free" discharge over a weir, and the equation is obviously

$$q = \frac{2}{3}\, c_w \sqrt{2g}\, b_w\, (H - B)^{\frac{3}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (66)$$

Coming now to the vents, we see that the discharge is similar to that over a drowned weir, for from the height x down to the height R the discharge is "free," while throughout the height R the head of pressure is constant, and the discharge is similar to that of a submerged sluice. To the whole discharge the "approach" coefficient C_s has to be applied. Thus the discharge of the vents is

$$q = C_s \cdot c_s \sqrt{2g}\, b_s \left\{ \frac{2}{3}(x - R)^{\frac{3}{2}} + R\,(x - R)^{\frac{1}{2}} \right\}$$

For our present purpose it will be most convenient to write this equation in the following form

$$\left(x + \frac{R}{2}\right)(x - R)^{\frac{1}{2}} = \frac{q}{\frac{2}{3} c_s \sqrt{2g}\, b_s\, C_s} \quad \ldots \quad \ldots \quad (67)$$

In this equation the value of C_s has to be obtained from the velocity of approach Table XVIII, using as the index-value

$$\frac{ca}{\Omega} = \frac{c_s\, b_s\, x}{b_x x} = \frac{c_s\, b_s}{b_x}$$

This completes the first case.

The second case occurs when the level in the intermediate space has risen above the crest of the weir, but not as high as the top of the vents, *i.e.*, when x is greater than B and less than D (see *fig.* 32).

The weir discharge is now "drowned" and its equation is consequently

$$q = c_w \sqrt{2g}\, b_w \left\{ \frac{2}{3}(H - x) + (x - B) \right\} (H - x)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (68)$$

The discharge through the vents is clearly of the same form as equation (67) above.

In the the third case, the level in the intermediate space has risen above the top of the vents, while the tideway level is still below that point. That is, x is greater than D, and R is less than D (see *fig.* 33).

Here the discharge over the weir is identical in form with equation (68), but the discharge through the vents takes the form of that of a partially-submerged sluice, as follows:—

$$q = C_s c_s \sqrt{2g}\, b_s \left\{\frac{2}{3}(x - R)^{\frac{3}{2}} - \frac{2}{3}(x - D)^{\frac{3}{2}} + R\,(x - R)^{\frac{1}{2}}\right\}$$

Which may be written as follows

$$\left(x + \frac{R}{2}\right)(x - R)^{\frac{1}{2}} = (x - D)^{\frac{3}{2}} + \frac{q}{\frac{2}{3} c_s \sqrt{2g}\, b_s\, C_s} \quad \ldots \quad (69)$$

In this case the value of C_s is obtained from Table XVIII, using the index-value $\frac{ca}{\Omega} = \frac{c_s\, b_s\, D}{b_x\, x}$.

Lastly, the fourth case occurs when both x and R are greater than D (see *fig.* 34).

Here the weir-discharge is evidently the same as in the second and third cases, and its equation is given in (68) above. The vent discharge occurs as through a completely-submerged sluice, under the head $(x - R)$, and the approach coefficient C_s has to be applied. The equation is as follows:—

$$q = C_s\, c_s \sqrt{2g}\, b_s\, D\,(x - R)^{\frac{1}{2}}$$

and it may be written, for convenience, as follows:—

$$R = x - \left(\frac{q}{c_s \sqrt{2g}\, b_s\, D\, C_s}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad (70)$$

where the value of C_s is obtained from Table XVIII, using the index-value

$$\frac{ca}{\Omega} = \frac{c_s\, b_s\, D}{b_x\, x}$$

I will now indicate how these equations may be solved in practice. The value of H is given, and we have to find what values of q correspond with any given values of R; that is, we have to find the discharge of the sluice at any given state of the tide. In order to do this, we have to find also the value of x, but this value is only required as a stopping stone to the values of q and R. The solution has to be obtained by trial, as follows. First, select some convenient value of x. Then calculate the value of q from the weir-equation, *i.e.*, from whichever of equations (66) or (68) is applicable. Then find the value of R from the vent-equation (67), (69), or (70), whichever is applicable. This operation must be carried out by trial methods, except in the case of equation (70). We now have one complete set of corresponding values of x, q, and R. Next, selecting a second convenient value of x, a second set of values can be obtained, in the same manner. When several sets of values have been thus determined, a curve can be plotted, to any convenient scale, with the values of R as abscissæ and of q as ordinates. The values of x, as I have just mentioned, are not required any further. When we have plotted this curve, we can then scale off from it the values of q corresponding with any particular value of R required.

Now it must be admitted that this solution is extremely cumbrous and laborious, and although it might be necessary to use it if we wished to make out a complete "discharge table," showing the discharges of the sluice at all heights of the gauges, yet we want something much simpler for practical work in designing, where several trial designs might have to be rejected before a suitable one was arrived at. As a first step in the direction of simplicity, we may take the following method, which I will call the "first approximation." Taking first the case where the weir discharges "free" at low water, let R_0 (see *fig.* 35) be the height of the tideway level at the moment when the weir begins to be "drowned," *i.e.*, when x is equal to B; and let t_0 be

the time the tide takes in rising from O to R_o and t_1 the time of rise from R_o to H. Let q_o be the discharge when the tideway stands at the height R_o. Then since the weir discharges "free" when the tideway is below R_o, the discharge throughout the time t_o is uniformly equal to q_o. We now come to the approximation, which consists in assuming that the rate at which x increases from B to H is proportional to the rate at which R increases from R_o to H; that is,

$$\frac{x-B}{H-B} = \frac{R-R_0}{H-R_0}$$

This gives us the relation $x = B + \dfrac{H-B}{H-R_o}(R-R_o)$ ... (71)

We can now proceed as follows. First find the value of q_o from equation (66). Next, using this value of q, find R_o from equation (67) or (69), whichever is applicable. The form (69) may be taken as the general form, on the understanding that when x is less than D the term $(x-D)$ vanishes. In this case we have to write B for x, so that the equation takes the form—

$$\left(B+\frac{R_o}{2}\right)(B-R_o)^{\frac{1}{2}} = (B-D)^{\frac{3}{2}} + \frac{q_o}{\frac{2}{3}c_s\sqrt{2g}\,b_s\,C_s} \quad \ldots \quad \ldots \quad (72)$$

The value of C_s is obtained from Table XVIII, using the index-value $\dfrac{ca}{\Omega} = \dfrac{c_s b_s D}{b_x B}$ when B is greater than D, and $\dfrac{ca}{\Omega} = \dfrac{c_s b_s}{b_x}$ when B is less than D. Now the value of R_o has to be found by trial from this equation (72), and when this has been done the value of q for any given value of R can be found from equations (71) and (68).

This completes the solution in the case when the weir discharges "free" at the beginning. It may, however, happen that the weir is "drowned" from the beginning, at low water, owing to the smallness of the vents or the lowness of the breast-wall. In this case x_o is not equal to B, and our method has to be modified, as follows. We now have t_o equal to *zero* and R_o *zero*, and the value of x_o has to be found from equations (67) and (68), which take the following forms (since $R_o = 0$)

$$q_o = \frac{2}{3} C_s c_s \sqrt{2g}\, b_s\, x_o^{\frac{3}{2}}$$

and $q_o = \dfrac{2}{3} c_w \sqrt{2g}\, b_w \left\{ (H-x_o) + \dfrac{3}{2}(x_o - B) \right\} (H-x_o)^{\frac{1}{2}}$

Dividing, the following expression is obtained,

$$\left(\frac{H-x}{x_o}\right)^{\frac{1}{2}} = \frac{C_s c_s b_s}{c_w b_w} \frac{x_o}{H-x_o+\frac{3}{2}(x_o-B)} \quad \ldots \quad \ldots \quad (73)$$

From this equation (73) the value of x_o can be found by trial. The value of c_s in this case would be perhaps more appropriately given by the value of C_w in Table XVIII, using the index-value $\dfrac{ca}{\Omega} = \dfrac{c_s b_s}{b_x}$

Now that we have thus determined the values of x_o and q_o, we can assume as before that the increase of x is proportional to that of R, thus:

$$\frac{x-x_0}{H-x_o} = \frac{R}{H}$$

That is, $x = x_o + R\left(1 - \dfrac{x_o}{H}\right)$ ... ... (74)

We can now find any other required value of q from equation (68)

This completes the first approximation. It will be seen that the laborious trial methods are not altogether avoided, as they have to be employed in determining R_o or x_o. There is, however, a considerable gain in simplicity over the accurate method. The trial method may be completely avoided by using another method, which I will call the second approximation, and which

is as follows. When the weir discharges "free" at low water, the value of R_o has, in the first approximate method, to be found by trial from equation (72). To avoid this, write $\frac{3}{2}B$ instead of $\left(B + \frac{R_o}{2}\right)$, and the equation then becomes

$$R_o = B - \left\{ \frac{2}{3} \frac{(B-D)^{\frac{3}{2}}}{B} + \frac{q_o}{(C_s\, c_s\, \sqrt{2g}\, b_s\, B)} \right\}^2 \quad \ldots \quad (75)$$

The value of C_s is obtained from Table XVIII, using the index-value $\frac{ca}{\Omega} = \frac{c_s\, b_s\, D}{b_x\, B}$ where B is greater than D, and the value $\frac{ca}{\Omega} = \frac{c_s\, b_s}{b_x}$ when B is less than D. The value of q_o is obtained from equation (66) as before: and any other values of x and q can be found from equations (71) and (68). This completes the solution when the weir discharges "free" at low water.

When the weir is "drowned" at low water, we had, in the first method, to find the value of x_o by trial from equation (73). To avoid this, in the second approximation we attribute to x_o, *on the right-hand side only* of equation (73), the value $\frac{B+H}{2}$, and the equation then becomes

$$\left(\frac{H-x_o}{x_o}\right)^{\frac{1}{2}} = \frac{C_s\, c_s\, b_s}{c_w\, b_w} \cdot \frac{2}{5}\, \frac{H+B}{H-B}$$

That is,

$$x_o = \frac{H}{1 + \left(\frac{2}{5}\, \frac{C_s\, c_s\, b_s}{c_w\, b_w}\, \frac{H+B}{H-B}\right)^2} \quad \ldots \quad \ldots \quad \ldots \quad (76)$$

The index value for C_s being $\frac{ca}{\Omega} = \frac{c_s\, b_s}{b_x}$ as in equation (73).

This determines x_o, and the rest of the solution is the same as in the first approximation.

There is a third approximation, less accurate than either of the preceding methods, but still sufficiently near the truth in many cases, and it consists in calculating the discharge over the weir as if the vents did not exist. This is equivalent to assuming that x is always equal to R, and it follows, of course, that $x_o = R_o = B$. When the weir is "free," *i.e.*, when $R \lesseqgtr B$, the discharge is given by equation (66). When the tide rises above the crest of the weir, *i.e.*, when $R \gtreqless B$, the discharge is

$$q = c_w \sqrt{2g}\; b_w \{\tfrac{2}{3}(H-R) + (R-B)\}\, (H-R)^{\frac{1}{2}} \quad \ldots \quad (77)$$

We now have four different methods, of gradually increasing simplicity, of determining the discharge of our sluice, and the best way of comparing the results of each method is to take an actual example. We will select the dimensions of a small sluice lately designed for the 24-Parganas embankment, which give us the following data:—

$$H = 10'\,;\; D = 8'\,;\; B = 6'\,;\; b_w = 18'\,;\; b_x = 13'\,;\; b_s = 10'.$$

The ordinary coefficient for a weir is $\frac{2}{3}\, c_w \sqrt{2g} = \frac{10}{3}$, so that the value of c_w is ·62 and $c_w \sqrt{2g} = 5$. As regards the result, there is very little contraction, as the floor is flat, the wing-walls converge gradually, and the pier is fitted with a cut-water. The coefficient must therefore be at least as high as $c_s \sqrt{2g} = 6{\cdot}5$; that is, $c_s = {\cdot}81$. With these figures the equations are as follows:—

Equation (66) is

$$q = \frac{2}{3} \times 5 \times 18\, (10-6)^{\frac{3}{2}} = 480 \text{ c. f. s.} \quad \ldots \quad \ldots \quad (66a)$$

In equation (67), the index-value for determining the value of C_s from Table XVIII is $\frac{ca}{\Omega} = \frac{{\cdot}81 \times 10}{13} = {\cdot}609$; and the corresponding value of C_s is 1·202. Equation (67) now becomes—

$$\left(x + \frac{B}{2}\right)(x-R)^{\frac{1}{2}} = \frac{q}{54{\cdot}078} \quad \ldots \quad \ldots \quad \ldots \quad (67a)$$

Equation (68) becomes—

$$q = 5\times18 \left\{ \frac{2}{3}(10-x) + (x-6) \right\} (10-x)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad (68a)$$

and (69) is as follows:—

$$\left(x + \frac{R}{2}\right)(x-R)^{\frac{1}{2}} = (x-8)^{\frac{3}{2}} + \frac{q}{43\cdot333\ C_s} \quad \ldots \quad \ldots \quad (69a)$$

where the index-value for finding C_s from Table XVIII is

$$\frac{ca}{\Omega} = \frac{4\cdot985}{x}$$

The remaining equation (70) is

$$R = x - \left(\frac{q}{520\ C_s}\right)^2 \quad \ldots \quad \ldots \quad \ldots \quad (70a)$$

where the index-value for C_s is $\frac{ca}{\Omega} = \frac{4\cdot985}{x}$

In obtaining the solution by the accurate method, we must first see whether the weir will discharge "free" at low water. Using equations (66a) and (67a) and writing $R = 0$, we have

$$x_o^{\frac{3}{2}} = \frac{480}{54\cdot678}$$

whence we obtain for x_o the value 4·256. Thus x_o is less than B, and the weir discharges "free."

Next, find the value of R_o, the elevation of the tideway when "drowning" begins. In equation (67a) write $x = B = 6'$ so that

$$\left(6 + \frac{R}{2}\right)(6-R)^{\frac{1}{2}} = 8\cdot779$$

It is found by trial that this is satisfied by the value $R = 4\cdot924$.

Next, find the discharge as "drowning" progresses, from (68a)

Taking first $x=7$, we have from equation (68a)

$$q = 5 \times 18 \left\{ \frac{2}{3} \times 3 + 1 \right\} \sqrt{3} = 467\cdot65$$

and substituting this value in (67a)

$$\left(7 + \frac{R}{2}\right)\left(7-R\right)^{\frac{1}{2}} = \frac{467\cdot65}{54\cdot678} = 8\cdot553$$

This is found by trial to be satisfied by the value $R = 6\cdot289$

Taking another value of x, say $x = 8$, the values obtained from equations (68a) and (67a) are $q = 424\cdot27$ and $R = 7\cdot566$.

Again, when $x = 9$ feet, the value of q from (68a) is 330. In this case the tide-way level will be above the top of the vents, so that equation (70a) will be the proper one to use. The index-value for C_s is $\frac{ca}{\Omega} = \frac{4\cdot985}{9} = \cdot554$, and the corresponding value of C_s from Table XVIII is 1·202. Equation (70a) now becomes

$$R = 9 - \left(\frac{330}{520 \times 1\cdot202}\right)^2 = 8\cdot721$$

Taking one more value of x, *viz.*, $x=9\cdot8$, the value of q from (68a) is,

$$q = 90 \left\{ \frac{2}{3} \times \cdot2 + 3\cdot8 \right\} \sqrt{\cdot2} = 158\cdot32$$

For the solution of (70a) we have $\frac{ca}{\Omega} = \frac{4\cdot985}{9\cdot8} = \cdot509$, so that the value of C_s from Table XVIII is 1·163, and equation (70a) becomes

$$R = 9\cdot8 - \left(\frac{158}{520 \times 1\cdot163}\right)^2 = 9\cdot731$$

All these values are shown in column (1) of the comparative Table XIX.

This will be quite a sufficient number of values to calculate, as any other intermediate values can be scaled off from a curve plotted with the above values of R as abscissæ and of q as ordinates. The principal object, however, of the calculations we are now making is to compare the results obtained by the use of the approximate methods described above with those of the exact

method. Having got our results by the exact method we will now see what values of x and q will be given by the approximate methods, for the same values of R. I need not occupy any more time in tracing the arithmetical steps of the process, but will briefly mention how the results are obtained.

Coming to the first approximation, the value of R_0 is obtained in exactly the same way as in the exact method and is of course the same as above, *viz.*, $R_0 = 4{\cdot}924$. Now taking equation (71), and using this value of R_0, and substituting in it successively the same values of R as we obtained by the first method, we can calculate the corresponding values of x. The values of q corresponding with these values of x are calculated from equation (68*a*). These results of the first approximate method are shown in column (2) of Table XIX.

We next come to the second approximation. The value of R_0 is got from equation (75). We have first to obtain the value of C_s from Table XVIII, which is given by using the index value $\frac{ca}{\Omega} = \frac{{\cdot}81 \times 10}{13} = {\cdot}623$ (since B is less than D), the corresponding value of C_s being 1·28. From equation (75) we get the value $R_0 = 5{\cdot}079$, which is a very fair approximation to the true value 4·924 obtained above. The remaining values of x and q are calculated from equations (71) and (68*a*) exactly as before, and the results are shown in column (3) of Table XIX.

Lastly, we come to the third and roughest approximation, in which it is assumed that x is always equal to R, and consequently when "drowning" begins $x_0 = R_0 = B = 6'$. The values of q are obtained from equation (77), which becomes

$$q = 60\left(1 + \frac{R}{2}\right)\sqrt{10 - R} \qquad \ldots \qquad \ldots \qquad \ldots \quad (77a)$$

The results are shown in column (4) of Table XIX. An examination of this table shows you that in this example the approximate methods give discharges, as a rule, in excess of the truth, especially the 3rd approximation; and that the results of the 2nd method are practically as close to the truth as those of the 1st.

TABLE XIX.

Comparative Statement.

	1	2	3	4
	Exact method.	First approximation.	Second approximation.	Third approximation.
$R_0 =$ ...	4·924	4·924	5·079	6·000
$x_0 =$ ...	6·000	6·000	6·000	6·000
$q_0 =$ ...	480	480	480	480
$R =$ ...	6·289	6·289	6·289	6·289
$x =$ ...	7·000	7·076	6·984	6·289
$q =$ ...	468	466	468	479
$R =$ ...	7·566	7·566	7·566	7·566
$x =$ ...	8·000	8·082	8·022	7·566
$q =$ ...	424	419	423	448
$R =$ ...	8·721	8·721	8·721	8·721
$x =$ ...	9·000	8·992	8·960	8·721
$q =$ ...	330	331	335	364
$R =$ ...	9·731	9·731	9·731	9·731
$x =$ ...	9·800	9·788	9·781	9·731
$q =$ ...	168	163	165	183
$R =$ ...	10·0	10·0	10·0	10·0
$x =$ ...	10·0	10·0	10·0	10·0
$q =$ ...	0	0	0	0

VII.—PRACTICAL DESIGN OF DRAINAGE SLUICES.

In actually designing sluices of this type, the most practicable method of proceeding will be to work in the first instance by the third approximation, design the sluice completely by this method, and then test the result by the more exact methods and correct the design if necessary. This simplifies the design enormously, because the third approximation, as we have seen, consists in regarding the structure as if it consisted simply of a weir, without any vents below it, and consequently we are enabled to apply the results which I pointed out to you in Sections III and IV of these Lectures. We know from the tidal curve the number of hours during one tide that the weir discharges "free" (which of course occurs, on the present assumption, while the tide-level is below the height B), and we may call this t_o. Let us suppose, in this case, that the tide rises considerably above the height H, so that the rate of rise from B to H is uniform; and let us use t_1 to represent the number of hours during which the tide is either rising from B to H or falling from H to B. The time during which the tide stands above the level H may be represented by t_2, and we then have $t_o + t_1 + t_2 = 13$. Writing q_m for the true mean discharge during the time t_1, q_o for the maximum discharge which occurs during the time t_o, *i.e.*, while the weir is discharging "free;" and noting that the discharge during the time t_2 is zero, we have, as the true mean discharge during the whole tide, which we will represent by q_{mm}

$$q_{mm}\,(t_o + t_1 + t_2) = q_o t_o + \frac{q_m}{q_o} q_o t_1 + 0$$

That is, $$q_{mm} = \frac{t_o + \frac{q_m}{q_o}\, t_1}{13}\, q_o \qquad \ldots \qquad \ldots \qquad \ldots \quad (78)$$

Suppose now it is required to design a sluice with a breast-wall of such capacity as to lower the flood-level over an area of M square miles at the rate of r inches in 24 hours. We know, as a matter of arithmetic, that a flow-off of 1 inch in 24 hours from 1 square mile is equal to a continuous discharge of 26·89 c. f. s. lasting for 24 hours, so that we have for the value of q_{mm}

$$q_{mm} = 26{\cdot}89\ Mr$$

and substituting this value in equation (78), we have for the value of q_o

$$q_o = \frac{350\ Mr}{t_o + \frac{q_m}{q_o}\, t_1} \qquad \ldots \qquad \ldots \qquad \ldots \quad (79)$$

The breast-wall must be of sufficient width to pass this discharge when discharging "free." The discharge, of course, depends on the depth of water passing over the crest of the wall, *i.e.*, on the quantity $(H—B)$, which must be fixed arbitrarily, at such a value that the velocity through the sluice, and the hammering action of the water on the floor, may not exceed a safe limit. For instance, suppose we consider, from practical experience, that the depth of overflow should not exceed 4 feet, then the breast-wall would be built up to a height 4 feet below the flood-level. Some allowance also should be made for the actual height of the wall B, because it is clear that a small quantity of water falling from a great height would cause more hammering action on the floor than a greater depth of overflow with a low fall; but, on the other hand, the scouring action below the sluice would in any case be greater with the greater depth of overflow. In fact, this is a matter which must vary with circumstances, and can only be decided by experience. Calling the depth of overflow $(H—B)$, we have

$$q_o = \frac{10}{3} b_w\, (H—B)^{\frac{3}{2}} \qquad \ldots \qquad \ldots \qquad \ldots \quad (80)$$

and combining this with (79), we have for the value of b_w

$$b_w = \frac{105\ Mr}{(t_o + \frac{q_m}{q_o}\, t_1)\,(H—B)^{\frac{3}{2}}} \qquad \ldots \qquad \ldots \quad (81)$$

The value of $\frac{q_m}{q_0}$ appropriate to the case under consideration must be selected from the results I have given you.

The foregoing results may be exemplified as follows:—

Suppose it is desired to drain a swamp whose level is 7 feet above mean sea level, by means of a sluice discharging into a tideway where the mean typical tidal curve has a range from 0 feet above to 3 feet below mean sea level. The drainage water is led from the swamp to the sluice by an open channel 6 miles long. The area to be drained is 30 square miles, and the water over this area has to be lowered at the rate of $\frac{3}{4}''$ in every 24 hours. Now in this case the levels which are known are those of the swamp and of the tideway, and we have to decide on the dimensions of the channel, as well as those of the sluice. Now you know that the discharging capacity of a channel depends partly on its breadth and depth and partly on its surface-slope, so that to carry a given discharge we can design a narrow channel with a steep slope or a broad channel with a flat slope. So far as the channel is concerned, the former is of course the more economical, but in the present case we have also to remember that a steep slope in the channel involves a lower level at its tail, *i.e.*, just above the sluice, and a smaller "head" of discharge over the sluice, which necessitates a bigger sluice. In other words, the steeper we make the slope, the cheaper will be the channel, and the dearer will be the sluice; and we have to decide which will be the most economical slope to give the channel. This can only be done by trial in each case, but, as a rule, it will probably be found more effective to design the sluice so as to give rather a flat slope to the channel, and thus keep up the level of the water above the sluice as high as possible, as this will enable the sluice to discharge for a greater number of hours during the day. Suppose in the present case we decide to allow for a surface-slope of 6 inches per mile in the channel. The dimensions of the channel may be considered presently, but so far as the sluice is concerned we have to notice that the drop of level in the 6 miles from the swamp to the sluice is 3 feet, so that the level of the water in the channel inside the sluice is 4·00 above mean sea level. Suppose, again, we decide, from previous experience of similar sluices, that 4 feet depth of water passing over the breast-wall is as much as is desirable. Then the crest of our weir is fixed at the level 0·00.

Fig. (36) represents the tidal curve, the abscissæ being times and the ordinates the respective elevations of the tidal surface at these times. We can now proceed to the design of the sluice; and first we will accept the third approximation as sufficiently accurate. That is, we will assume the discharge to occur as over a weir, with no vents in front of it. Since the crest of the weir is at + 0·00, the discharge will be "free" while the tide level is below + 0·00, that is during the times represented in fig. (36) by E F and K L. This corresponds with the t_0 of equation (78). We now have to remember that, as the tide rises, and backs up the discharge over the weir, the water at the lower end of the channel will rise at the same time. The discharge will be gradually reduced until the tide rises to nearly the level of the swamp. The height at which cut-off will actually occur, and the discharge cease, is uncertain, but for the purposes of this calculation we may assume the height to be + 0·00 and the rate of rise may be taken as uniform. The time t_1 is thus given by the distances FG and HK in fig. (36). The time during which discharge entirely ceases (t_2) is of course GH, while the tide is above the level 6·00. In section IV(b) of these Lectures I have shewn you how to calculate the actual value of the ratio q_m/q_0 for a channel and sluice of given dimensions. As a preliminary trial, in order to arrive at suitable dimensions, we may assume that value to be ·9. We can now obtain the value of b_w from equation (81), since we have $M = 30$; $r = \frac{3}{4}$; $t_0 = 4$; $t_1 = 4\cdot6$; $(H-B) = 4$. Thus $b_w = \frac{105 \times 30 \times \frac{3}{4}}{(4 + \cdot9 \times 4\cdot6) \times 4^{\frac{3}{2}}} = 36\cdot3$ or say 37 feet.

That is, the weir must be at least 37 feet wide, and, with a depth of 4 feet of water passing over it, the waterway will be ($4 \times 37 =$) 148 square feet. The vents should be designed so as to have at least this waterway, and it will be amply given by four vents each 5′ wide by 8′ high.

We can, if so desired, obtain the other quantities q_{mm} and q_o. Thus q_{mm}, the equivalent mean discharge throughout the whole time, is given by $q_{mm} = 26{\cdot}89$ $Mr = 26{\cdot}89 \times 30 \times \frac{3}{4} = 605$ c. f. s. and q_o, the "free" discharge of the weir, will be given by equation (79) thus—

$$q_o = \frac{350 \times 30 \times \frac{3}{4}}{4 + {\cdot}9 \times 4{\cdot}6} = 968 \text{ c. f. s.}$$

Supposing now we wish to work by the more accurate method which we have called the "second approximation," we shall have, as before, the level of the water above the sluice at $+ 4{\cdot}00$, and the crest of the weir at $+ 0\ 00$. The level of the floor of the sluice depends on practical considerations connected with the nature of the site, but it should as a general rule be at least as low as the level of lowest low water in the tideway. Suppose the floor to be at $- 4{\cdot}00$. Then we have $H = 4 + 4 = 8$ feet; $B = 4$ feet. Now looking at equation (76), it is seen that the solution involves dimensions (b_s and b_w) which have not been determined. It is therefore necessary to work out the design at first by the third approximation, as has been done above, and afterwards correct it, if necessary, by the more accurate methods. Designing the sluice with the dimensions calculated above, we have—

$$b_w = 37\,;\ c_r = {\cdot}62\,;\ b_s = 20\,;\ b_x = 29\,;\ c_s = {\cdot}81.$$

We have to find the value of x_o from equation (76). The index value for C is $\frac{ca}{\Omega} = {\cdot}56$, and the value of C_s from Table XIV is $1{\cdot}208$. Hence, from equation (76)

$$x_o = \frac{8}{1 + \left(\frac{2}{5}\,\frac{1{\cdot}208 \times 16{\cdot}2 \times 12}{{\cdot}62 \times 37 \times 4}\right)^2} = 3{\cdot}906$$

Now the value for x_o assumed in the third approximation (above) was $4{\cdot}00$; that is 4 feet above the floor, *i.e.*, at the level $+ 0{\cdot}00$. If we draw the line AB in fig. (36) at the height $- {\cdot}094$ instead of $+ 0{\cdot}00$ and again calculate the quantity $(t_o + \frac{q_m}{q_o} t_s)$, the difference will be so small as not to appreciably affect the result of equation (81).

Let us see now what difference it would have made if the error in the value of R_o, by the third approximation, had been as much as 1 foot. That is, suppose that drowning begins when the tide is at $- 1{\cdot}00$ instead of $+ 0{\cdot}00$. Then in fig. (36) drawing the lines A′B′, A′F′, B′K′, we find by measurement that $t_o = 3{\cdot}1$, $t_s = 5{\cdot}5$, and the value of $t_o + \frac{q_m}{q_o} t_s$ is $8{\cdot}05$ instead of $8{\cdot}14$ as it was before. The result of equation (64) will now be

$$b_w = \frac{8{\cdot}14}{8{\cdot}05} \times 36{\cdot}3 = 36{\cdot}7$$

instead of $36{\cdot}3$. The error is quite inappreciable, and is already covered by our selecting the dimension as 37 feet.

In fact, we may as a rule accept the third approximation as sufficiently accurate for the present purpose, though it would always be as well to check the results by the more exact methods.

We may now return to the design of the channel. In Section IV(*b*) of these Lectures I have described to you an accurate method of calculating the mean discharge of a combined channel and sluice throughout the whole tide. Before, however, that method is employed, we have to select, as a preliminary measure, certain definite dimensions for the channel and sluice, based on approximate calculations; and in selecting the dimensions of the channel, there is an important point to bear in mind, viz., that the duty of the channel is to "feed" the sluice; that is, the capacity of the channel must be such that it shall be capable of conveying towards the sluice *whatever* quantity the sluice is required to discharge. It is of no use designing a sluice to discharge 1,000 c. f. s. at low water, if the channel is incapable of carrying more than (say) 600 c. f. s.; the sluice can only discharge the water that is brought down to it, and if it is not kept fully supplied the rate of discharge must be reduced. This is a mistake that is very liable to occur, and should be carefully guarded

against. In the present example, the maximum discharge of the sluice, occurring at low water, is 968 c. f. s., and the mean discharge, while drowning is in progress, is ($\cdot 9 \times 968 =$) 871 c. f. s. Now suppose the channel were designed so that at its greatest capacity it was only capable of carrying the mean discharge of 871, or, still worse, the mean discharge during the whole time, viz., 605 c f. s., there would be a considerable loss of efficiency, the extent of which will perhaps be best shown graphically.

In fig. (37) measure the times as abscissæ along CB, and let the gradually decreasing "head" be represented by the ordinates of the line AB, and the discharges by the ordinates of the curve EDB. Neglect, for the present, the favourable effect produced on the discharges by the increased carrying capacity of the channel at increasing depths. Then the total discharge during the time that the weir is being "drowned" is shown by the area of the curve EDBCE, and the mean discharge, being $\frac{4}{5}$ths of the maximum, is represented by the ordinate MD. Now, if the channel can only carry the *mean* discharge, it follows that all higher discharges are prevented, and the total discharge is represented by the area CFDBC; or, in other words, the total discharge is reduced by the area EFD.

When the discharge occurs through a submerged sluice, the mean discharge is $\frac{2}{3}$rds of the maximum, and the curve EDB becomes a parabola, while the proportionate reduction of discharge is greater.

If the favourable effect of the deepening channel is taken into consideration, the curve of discharges, in the case of a weir, will be somewhat of the character shown in the diagram given as Appendix X of the example which forms Section IX(d) of these Lectures. In this case the error is less. The principle, however, is well exemplified, in any case, that the channel should be capable of carrying the *maximum* discharge which the sluice is required to pass.

In the present example the maximum discharge is 968 c. f. s., and the slope is 6′ per mile. Using Kutter's formula with N = 0·025 and side-slopes one to one, the required discharge would be given by a depth of 8 feet and a bed-width of about 51 feet.

It will be useful to enquire what effect on the discharge of the sluice is produced by the velocity of approach in the channel leading to the sluice. The discharge, it will be remembered, takes place as over a weir; and the "virtual" area of waterway over the weir is $c_w\ b_w\ (H-B)$, that is $= \cdot 62 \times 37 \times 4 = 91 \cdot 76$ square feet. The area of waterway in the channel itself is $(51+8)\ 8 = 472$ square feet, so that the index-value for use in Table XVIII is $\frac{ca}{\Omega} = \frac{92}{472} = \cdot 195$ and the corresponding value of C_v is about 1·02. This small increase of 2 per cent. is in our favour, and we need not alter the design, but may accept it as giving a small increase of efficiency beyond the calculated result.

VIII.—DISCHARGE OF OPEN CHANNELS WHEN THE SURFACE IS NOT PARALLEL TO THE BED.

I have already mentioned that the cheapest and simplest means of conveying large quantities of water is to excavate open channels in the ground, along which the water flows under the influence of gravity. We now have to consider in detail the capacity of these open channels; that is, we have to find an expression which will inform us what volume of water will flow past a given point in a given time, in terms of the breadth, depth, and shape of cross-section of the channel, and of its longitudinal inclination or "slope"; and we also have to take into consideration the material of which the bed and sides of the channel are composed, as this has an important effect on the result, owing to the action of fluid friction. Let us take first the simplest possible case, and as you will find it dealt with in your text-books I will not dwell on it long. I mean the case where, looking at a longitudinal section of the stream, the surface of the water is parallel to the bed of the canal. We have here two opposing forces, viz., (1) the force of gravity acting on the water, and causing it to virtually *fall* along the canal, in the direction of the longitudinal slope; and (2) the force of fluid friction, which, acting on the water in the opposite direction, resists this tendency to fall. When once "steady" motion has become established these two forces are equal and opposite. The gravity force, *i.e.*, the weight of the water, has of course to be resolved into two components, one of which acts perpendicularly to the bed of the canal, and simply causes pressure on it, an effect which does not concern us at present; while the other acts parallel to the bed, and this is the force which is opposed by fluid friction. In estimating the magnitude of the force of friction, we have to be guided by experiments, which have shown that the force varies approximately as the extent or area of rubbing surface, and as the square of the velocity of flow. You will find the details worked out in your books, and I will simply quote the result, which is as follows:—

$$q = \omega u = \omega \sqrt{\frac{2g\, mi}{\zeta}}$$

where q is the discharge in cubic feet per second; u is the mean velocity in feet per second; ω is the area of cross-section of the stream; m is the hydraulic mean depth; i the sine of the angle which the bed of the canal makes with the horizontal; and ζ is the coefficient of friction.

The formula is used in this form by D'Arcy and Bazin, and the experiments made by them supply us with values of ζ applicable to various conditions. But the formula is very commonly used in another shape, and it is important to recognise the relation between the two shapes. Suppose we agree to represent the quantity $\sqrt{\frac{2g}{\zeta}}$, in the above formula, by a single symbol C. We then have

$$q = \omega u = \omega\, C \sqrt{mi}$$
$$\text{or, } u = C \sqrt{mi}$$

and in this form it is usually called the "Chezy" formula. Here we see that the coefficient of *friction* ζ has been abandoned, and the coefficient of *velocity* C used instead. In this latter shape the formula is used by Ganguillet and Kutter, whose exhaustive experiments have provided us with values of the coefficient suitable to almost every possible case. For convenience, they put the coefficient C in the form $100c$; and we now see that the correspondence between the coefficients of *friction* and *velocity* is as follows:—

$$C = 100c = \sqrt{\frac{2g}{\zeta}}$$

$$\text{and conversely } \zeta = \frac{2g}{C^2} = \frac{2g}{(100c)^2}$$

It is important to remember this relation, whenever we wish to compare the results of Bazin's coefficient with those of Kutter; or, in other words, whenever the coefficients of *friction* have to be compared with those of *velocity*. In what follows it will be necessary to go back to first principles, and employ the coefficient of friction ζ.

Another point I have to notice is that the formula as generally used contains the quantity m, the "hydraulic mean depth." Now there is no special virtue about this quantity in itself, it being simply a convenient abbreviation for the quotient $\frac{\text{area}}{\text{wetted perimeter}}$, that is, $\frac{\omega}{p}$; so that if we please we can use this detailed form instead of the abbreviation m. The formula then becomes

$$q = \omega \sqrt{\frac{2g\,\omega i}{\zeta p}}, \text{ or } q^2 = \frac{2gi}{\zeta p}\omega^3$$

and I shall have occasion presently to use it in this form.

I have, in a previous lecture, referred generally to the divergence of the conditions met with in practice from those dealt with in the text-books, and there is perhaps no more striking or important instance of this than the case I now propose to deal with in detail, viz., the flow of water in an open channel when the surface of the stream is not parallel to the bed of the channel. One very common cause of this want of parallelism is the existence of a weir or regulator in a stream. If a stream is flowing in a uniformly graded canal, and its surface is artificially raised at any place by a weir, the depth at the weir is increased above the normal depth for a considerable distance, perhaps for a good many miles, and as we proceed upstream from the weir the depth becomes less and less, until we arrive at a place where it again becomes practically normal, with the surface parallel to the bed. Theoretically the distance is infinite, there being no place above the weir where the stream absolutely regains its normal depth; but we are not concerned, from a practical point of view, with infinitely small variations of depth, and we can calculate, from the equations I shall presently give you, at what distance above the weir the depth of the stream varies by any given definite amount, however small, from the normal. It must be noted that the profile of the surface is a curve, and not a straight line, the curve being asymptotic to the straight line representing the normal surface. A similar increase of depth occurs when there is any sort of obstruction in the stream, as for instance a sudden contraction in the *width* of the stream. On the other hand, when a fall, or sudden drop, occurs in the bed of a stream, the depth is decreased below the normal, and as we proceed upstream it gradually increases until it becomes again very nearly normal, with the surface parallel to the bed. Near the drop, the decrease of depth reduces the area of cross-section of the stream, and consequently increases the velocity, as the discharge has to pass through a smaller cross-section. This increased velocity may be so great as to scour away the bed and banks of the canal, and to prevent this it is usual to counteract the effect of a fall by placing a weir on its crest, which is usually fitted with boards, to enable the surface of the upper reach to be regulated to any desired level.

Variations from surface parallelism are common in drainage schemes. Suppose a tract of country is flooded to a depth of 10 or 12 feet, the flood level being 50 feet above the sea, and it is desired to drain it into an outfall, whose surface is 30 feet above the sea, by an open channel 10 miles long. Here the surface slope is fixed for us, amounting to 20 feet in 10 miles, *i.e.* 2 feet per mile, and we can grade the bed of the channel at that slope. But you will see that, as the drainage progresses, and by the time that the flood level has been lowered by 10 feet, *i.e.*, to 40 feet above the sea, the surface slope will be only 10 feet in 10 miles, or *half* what it was at first. Here we have a bed slope of 2 feet per mile and a *mean* surface slope of 1 foot per mile (for, as I have just pointed out, the surface is a curve, and the slope is not uniform) forming a marked case of want of surface parallelism. You will readily see that when we have to deal with tidal outfalls, where the level of the outfall may vary 10 or 12 feet in a few hours, and never remains at the same level, this absence of parallelism always exists. This problem, of determining the discharge of a stream flowing with steady motion, when the inclination of the surface differs from that of the bed, can only be exactly solved by forming the equation of

motion in detail. The curve of the profile of the surface is determined by the differential equation of motion, and the integration of this equation gives us the difference of depth between any two places; or rather, in the form which the equation takes, it gives us the distance apart of any two cross-sections of given area, when a given disharge is flowing along the canal. This integral equation is necessarily complicated, and the only way in which I have been able to simplify matters is by calculating out tables, enabling you to find at a glance, in all cases likely to occur, the value of the very long and troublesome factor which occurs in the equation. You will notice that I have retained ω, the area of cross-section, as the variable, instead of the simple depth, and that I have introduced the quantity Ω representing the area of cross-section which *would* give the same discharge *if* the surface were parallel to the bed.

The differential equation is most conveniently formed in what is known as the "Eulerian" form, by estimating the changes which the external forces produce in the momentum of the fluid, considered with reference to a definite region in space. To do this we have to equate (i) the net rate of inflow of momentum into the region *plus* (ii) the external forces acting on the fluid in the region, to (iii) the rate of change of momentum in the region, and since the motion is "steady" this rate of change is *zero*.

Fig. (38) represents a longitudinal section of a portion of the canal, the bed being inclined to the horizon at an angle whose sine is i. The axis of x is taken as coinciding with the bed of the channel, and, as drawn, the depth of the water is increasing in the direction of motion.

Consider now the region A B C D, of length dx, and let ω represent the cross-sectional area of the stream at A B, and u its mean velocity past A B. Then the area of the section at C D is $\omega + \frac{d\omega}{dx}\,dx$ and the velocity $u + \frac{du}{dx}\,dx$.

The quantity of fluid which enters the region past the boundary A B in unit time is ωu, and the rate of inflow of momentum is $\frac{G}{g}\,\omega u^2$. The outflow of momentum past the boundary C D is $\frac{G}{g}\omega u^2 + \frac{G}{g}\,\frac{d}{dx}(\omega u^2)dx$, so that the net inflow of momentum is $-\frac{G}{g}\frac{d}{dx}(\omega u^2)dx$.

Next, we have to estimate the value of the external forces acting on the fluid, parallel to the bed. They are three in number, viz.—

(*a*) The component of the weight of the fluid, resolved parallel to the bed. The value of this is clearly $+\,Gi\omega dx$.

(*b*) Then there is the force of fluid friction, which acts in a direction opposite to the motion of the fluid, and is proportional to the rubbing-surface (pdx) and the square of the velocity (u^2). The value of it is $-\,\frac{G}{2g}\,\zeta\,pu^2dx$.

(*c*) Lastly, we have the resultant of the hydrostatic pressures on the boundary sections A B and C D. Now, if ω is the area of the section, y its depth, and γy the depth of the centre of gravity of the section below the surface, the pressure on the section is $G.\,\gamma y.\,\omega$. This is the pressure on A B; and the pressure on C D is—$\left\{G\gamma y\,\omega + G\,\frac{d}{dx}\,(\gamma y\,\omega)\,dx\right\}$; so that the resultant pressure is $-G\,\frac{d}{dx}\,(\gamma y\,\omega)\,dx$.

The canals we have to deal with are mostly of trapezoidal section (see *fig.* 39), and it will be found that for a section of this shape, of depth y, side-slopes σ to 1, surface breadth b, and bed-width b_0, the value of $\gamma\,y\,\omega$ is $(\frac{1}{2}b_0 + \frac{1}{3}\,\sigma y)\,y^2$, where $\omega = y\,(b_0 + \sigma y)$. From this last relation we have $\frac{d\omega}{dx} = (b_0 + 2\sigma\,y)\,\frac{dy}{dx}$ $= b\,\frac{dy}{dx}$, and we can now see that $\frac{d}{dx}\,(\gamma\,y\,\omega) = \frac{d}{dx}\,\left\{(\frac{1}{2}\,b_0 + \frac{1}{3}\,\sigma y)\,y^2\right\} =$ $\left(b_0 y + \sigma y^2\right)\frac{dy}{dx} = \omega\,\frac{dy}{dx} = \frac{\omega}{b}\,\frac{d\omega}{dx}$.

There is a rather convenient way of representing the value of γ for various sections, which I may mention in passing. Using the symbol β to

represent the ratio of the difference of the surface and bed-widths to the surface width, *i.e.*, writing β for $\frac{b-b_0}{b}$, and the value of γ being $\frac{b_0+\frac{2}{3}\sigma y}{2(b_0+\sigma y)}$, we can write γ in the form $\gamma = \frac{1}{2} \cdot \frac{1-\frac{2}{3}\beta}{1-\frac{1}{2}\beta}$

Now clearly in a rectangular section the surface and bed-widths are equal, and $\beta = 0$, so that the arithmetical value of γ is $\frac{1}{2}$, as it obviously should be.

A table can, if necessary, be constructed giving the values of γ for various values of β which may vary from 0 in a rectangular section to 1 in a triangular section. Cases where the surface is narrower than the bed are not likely to occur in practice.

Returning now to the differential equation, we can write it down completely, as follows :—

$$-\frac{G}{g}\frac{d}{dx}(\omega u^2)\,dx + G\,i\omega dx - \frac{G}{2g}\,\zeta\,pu^2dx - G\,\frac{\omega}{b}\frac{d\omega}{dx}\,dx = 0 \qquad (82)$$

Now, if we write for u its value $\frac{q}{\omega}$, q being the constant discharge of the stream, the equation will be expressed in terms of one variable, viz., ω. Remarking also that $\frac{d}{dx}(\omega u^2) = \frac{d}{dx}\left(\frac{q^2}{\omega}\right) = -\frac{q^2}{\omega^2}\frac{d\omega}{dx}$, and multiplying the whole equation by $g\frac{\omega^2}{G}$, it can be written in the form

$$gi\omega^3 - \frac{\zeta}{2}p\,q^2 - \frac{g}{b}\omega^3\frac{d\omega}{dx} + q^2\frac{d\omega}{dx} = 0 \quad \ldots \quad \ldots \quad (83)$$

or as follows

$$i\,dx = \frac{\frac{q^2}{g}b - \omega^3}{\frac{\zeta q^2}{2gi}p - \omega^3}\,\frac{d\omega}{b} \quad \ldots \quad \ldots \quad \ldots \quad (84)$$

This, then, is the differential equation of a stream when the surface is not parallel to the bed. Before going any further, it may be interesting to compare this with the ordinary expression applying to a parallel surface. The comparison, however, will only apply over such a short length of the stream that the surface is approximately a straight line. In fig. (40), let *ac* represent a unit length of the actual surface of the stream, and draw *ab* horizontal and *ad* parallel to the bed. Then the surface slope is the height *bc*, which we will represent by the letter s, and the rate of increase of depth, viz. $\frac{dy}{dx}$, is *cd*. The bed slope is of course i. We now have, when the depth is increasing in the direction of motion,

$$i = s + \frac{dy}{dx}; \qquad \text{whence } \frac{1}{b}\frac{d\omega}{dx} = \frac{dy}{dx} = i - s$$

and the differential equation (84) becomes

$$\left(\frac{q^2}{g}b - \omega^3\right)(i-s) = \left(\frac{\zeta q^2}{2gi}.p - \omega^3\right)i; \quad \text{that is, } q^2 = \frac{2gs}{\zeta - \frac{2(i-s)b}{p}}\,\frac{\omega^3}{p}$$

Now the ordinary expression, as we have seen above, if we substitute s the surface-slope for i the bed-slope, is

$$q = \frac{2gs}{\zeta}\frac{\omega^3}{p}$$

and comparing these two, we can write our differential equation in the form

$$\left.\begin{aligned} q^2 &= F \cdot \frac{2gs}{\zeta}\frac{\omega^3}{p} \\ \text{where } F \text{ has the value } &\frac{1}{1-\frac{2(i-s)}{\zeta}\frac{b}{p}} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (85)$$

That is, the true result is obtained by multiplying the approximate result by this factor F.

As an example, let us take a case where the value of c in Kutter's formula is ·85. Then

$\zeta = \frac{2g}{(85)^2} = \cdot 0089$. The value of $\frac{b}{p}$ will not, in most cases, differ much from 1.

Hence the value of F is $\dfrac{1}{1-\frac{2}{\cdot 0089}(i-s)}$

The values of F for a few values of $(i-s)$ are shown below:—

$(i-s)$ = (inches per mile) ...	0	6″	12″	24″
$F =$	1·000	1·0218	1·0445	1·0930

These results, of course, assume the constancy of ζ and $\frac{b}{p}$, which is not quite exact; but they are near enough to show that, in a short length, for small slopes, there is not much inaccuracy involved in working with the surface-slope.

A slightly closer result can be obtained by using the mean between the surface and bed slope, *i e.*, $\frac{i+s}{2}$, instead of the surface slope. The values of F just given must then be multiplied by the quantity $\frac{2s}{i+s}$.

When the depth is decreasing in the direction of flow, we shall have $\frac{dy}{dx} = -(s-i)$, and the result will be

$$F = \frac{1}{1+\frac{2}{\cdot 0089}(s-i)}$$

where s in this case is arithmetically greater than i.

When we have to deal with considerable lengths of canal, so that the profile of the surface is not approximately a straight line, it becomes necessary to integrate the differential equation (84). The variation of b and p, compared with that of ω, is taken to be so slow that they can be treated as mathematical constants.

Now returning to equation (84) and writing a^3 for $\frac{q^2}{g}b$ and Ω^3 for $\frac{\zeta q^2}{2gi}p$, the equation becomes

$$i dx = \frac{a^3-\omega^3}{\Omega^3-\omega^3}\frac{d\omega}{b}$$

but since we are considering the case where the depth increases down stream, ω is greater than Ω, and it will be more consistent to write

$$i dx = \frac{\omega^3-a^3}{\omega^3-\Omega^3}\frac{d\omega}{b} = \frac{d\omega}{b} + \frac{\Omega^3-a^3}{b}\frac{d\omega}{\omega^3-\Omega^3} \quad \ldots \quad \ldots \quad (86)$$

The integration of this, in general terms, is

$$x = \frac{\omega}{b} + \frac{\Omega^3-a^3}{6b\Omega^2}\left[\log_e\left\{\frac{(\omega-\Omega)^3}{(\omega^3-\Omega^3)}\right\} - 2\sqrt{3}\tan^{-1}\frac{2\omega+\Omega}{\sqrt{3}\Omega}\right] \quad \ldots \quad (87)$$

Now (see *fig.* 41) let ω_2 be the area of cross-section just above the weir, where the surface is raised, and ω_1 the area of another section distant L feet upstream from it. In the case of discharge into an outfall, ω_2 would be the section just above the outfall, and ω_1 the section just below the head-pool. Comparing the value of Ω with the ordinary expression for discharge where the surface is parallel to the bed, it is seen that Ω would be the area of cross-section far upstream, in the part undisturbed by the action of the weir. In a short stream, Ω may be looked on as that section which *would* result in the discharge q, *if* the surface and bed were parallel. In the present case ω_1 is less than ω_2, and Ω is less than either. Integrating now between the limits O and L, the equation becomes

$$i\,L = \frac{\omega_2}{b} - \frac{\omega_1}{b} + \frac{\Omega^3 - a^3}{6b\Omega^2}\left[\log_e\left\{\frac{(\omega_2-\Omega)^3\,(\omega_1^3-\Omega^3)}{(\omega_2^3-\Omega^3)\,(\omega_1-\Omega)^3}\right. - \right.$$

$$\left.2\sqrt{3}\left\{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\,\frac{\omega_2}{\Omega}\right) - \tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\,\frac{\omega_1}{\Omega}\right)\right\}\right] \quad \ldots \quad \ldots \quad (88)$$

For convenience in constructing labour-saving tables, this equation may be written in the following form :—

$$\frac{biL}{\Omega} = \frac{\omega_2}{\Omega} - \frac{\omega_1}{\Omega} + \frac{1}{6}\left(1 - \frac{a^3}{\Omega^3}\right)\left[\left\{\log_e \frac{\left(\frac{\omega_1^3}{\Omega^3} - 1\right)}{\left(\frac{\omega_1}{\Omega} - 1\right)^3} + 3{\cdot}464\tan^{-1}\left({\cdot}57736 + \right.\right.\right.$$

$$\left.\left.1{\cdot}15472\,\frac{\omega_1}{\Omega}\right)\right\} - \left\{\log_e \frac{\frac{\omega_2^3}{\Omega^3} - 1}{\left(\frac{\omega_2}{\Omega} - 1\right)^3} + 3{\cdot}464\;\tan^{-1}\left({\cdot}57736 + 1{\cdot}15472\,\frac{\omega_2}{\Omega}\right)\right\}\Bigg] \ldots (89)$$

Now, replacing the quantity $\frac{a^3}{\Omega^3}$ by its original value $\frac{2i}{\zeta}.\ \frac{b}{p}$, and denoting the long term in brackets by the symbol F_1, the equation becomes

$$\frac{biL}{\Omega} = \frac{\omega_2}{\Omega} - \frac{\omega_1}{\Omega} + \frac{1}{6}\left(1 - \frac{2i}{\zeta}\,\frac{b}{p}\right)F_1 \quad \ldots \quad \ldots \quad \ldots \quad (90)$$

The value of F_1 can be obtained from Table XX.

Next, in the case where the depth of the stream is decreasing in the direction of motion, the form of the equation, in general terms, is unchanged, but (see *fig.* 42) ω_1 is greater than ω_2 and the integration is taken, as before, from ω_1 to ω_2 so that the value of the $\frac{\omega}{b}$ term is $-\left(\frac{\omega_2}{b} - \frac{\omega_1}{b}\right)$ *i.e.*, $\frac{\omega_1}{b} - \frac{\omega_2}{b}$. Thus the equation becomes

$$i\,L + \frac{\omega_1}{b} - \frac{\omega_2}{b} = \frac{\Omega^3 - a^3}{6b\Omega^2}\left[\log_e \frac{(\Omega-\omega_2)^3\,(\Omega^3-\omega_1^3)}{(\Omega^3-\omega_2^3)\,(\Omega-\omega_1)^3} + \right.$$

$$\left.2\sqrt{3}\left\{\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\,\frac{\omega_1}{\Omega}\right) - \tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\,\frac{\omega_2}{\Omega}\right)\right\}\right] \quad \ldots \quad \ldots \quad (91)$$

As before, this may be written in the form

$$\frac{bi\,L}{\Omega} + \frac{\omega_1}{\Omega} - \frac{\omega_2}{\Omega} = \frac{1}{6}\left(1 - \frac{a^3}{\Omega^3}\right)\left[\log_e \frac{\left(1 - \frac{\omega_1^3}{\Omega^3}\right)}{\left(1 - \frac{\omega_1}{\Omega}\right)^3} + 3{\cdot}464\;\tan^{-1}\left({\cdot}57736 + \right.\right.$$

$$\left.\left.1{\cdot}15472\,\frac{\omega_1}{\Omega}\right)\right\} - \left\{\log_e \frac{\left(1 - \frac{\omega_2^3}{\Omega^3}\right)}{\left(1 - \frac{\omega_2}{\Omega}\right)^3} + 3{\cdot}464\tan^{-1}\left({\cdot}57736 + 1{\cdot}15472\,\frac{\omega_2}{\Omega}\right)\Bigg] \ldots (92)$$

Using the factor F_2 to represent this long term in brackets, the result can be expressed as follows:—

$$\frac{bi\,L}{\Omega} + \frac{\omega_1}{\Omega} - \frac{\omega_2}{\Omega} = \frac{1}{6}\left(1 - \frac{2i}{\zeta}\frac{b}{p}\right) F_2 \quad \ldots \quad \ldots \quad \ldots \quad (93)$$

where the value of F_2 is obtained from Table XXI.

This is the result, and you will see that it has worked out in a very inconvenient form. In most cases we are given the distance apart L of the two sections ω_1 and ω_2, and the levels of the surface at those sections, the unknown quantity which we have to determine being either the depth and breadth of the canal or the discharge q. This necessitates the use of trial methods which are tedious, even with the aid of these labour-saving tables; while without them the calculations would occupy an almost prohibitively long time. Probably the shortest way, in most cases, would be to assume, as a first trial value, the result obtained from the use of s, the mean surface slope, in the ordinary formula, and then correct the result by means of these equations. There are two points about the equations which must be borne in mind, to ensure their successful application. The first point is the variation in the quantities b and p, which have been treated as mathematical constants. You will see from the examples I propose to give you that the variation is, in most practical cases, very small, and the arithmetical mean value may safely be used without sensible error. What has to be considered is the variation of b and p between the limits ω_1, and ω_2. The variation of ω_1, or ω_2 from Ω does not, it should be noticed, affect the result; it is only the variation between ω_1 and ω_2 over which the limits of integration are taken, which concerns us. The second point is the variation of ζ the coefficient of friction. The value of this is computed from Kutter's coefficient of velocity, in the way I showed you above, and the latter coefficient must be obtained by reference to some of the many published tables on the subject. The best known are Jackson's tables, and Mr. Odling has also drawn up a convenient set, but the range they cover is rather limited. Sir Thomas Higham has also published some, and there are others. You will find from the examples that a small error in ζ makes a very small difference indeed in the result, so that no sensible error is likely to arise from this cause.

Example 1:—

As an example let us take the case of a canal whose bed-width is 100 feet, side-slopes 1 to 1, bed-slopes ·1 per 1,000, or very nearly 6 inches per mile; and length 4 miles. The depth of water at the head is 7 feet and at the tail 3 feet; find the discharge.

Now in this case we are not given the value of q, so the value of ζ cannot be exactly determined, and we must work with an approximate value of ζ, derived from some assumed value of q. This can be treated as a trial value, and the results can be corrected, if necessary, after we have found the value of q.

Now we see that the depth is increasing as we proceed up-stream, and that the normal depth, where the surface is parallel to the bed, would be greater than 7′, the depth at the head of the canal. Let us assume, for the purpose of calculating ζ, the normal depth to be 8 feet. Now from Jackson's Tables (pp. 292—293) we find that, with these data, the value of q is 1,967 and of C is ·858. The value of ζ is therefore $\frac{64{\cdot}4}{(85{\cdot}8)^2}$, that is, ·0087478. We now have

$$\frac{2i}{\zeta} = \frac{2 \times {\cdot}0001}{{\cdot}0087478} = {\cdot}022863.$$

Next, as regards the values of b and p, the surface width at a depth of 7 feet is 114 feet and at a depth of 3 feet b is 106 feet. The variation is small, and the mean value 110 feet may be taken. The value of p is 119·8 feet at the head and 108·5 feet at the tail. We can take the mean value 114 feet. Thus the factor $\frac{1}{6}\left(1 - \frac{2i}{\zeta}\frac{b}{p}\right)$ in equation (76) becomes $\frac{1}{6}$ (1 — ·022061), that is, ·16299. You will notice what very little difference the value of ζ makes in the result.

The values of ω_1 and ω_2 are 749 and 309 respectively and the equation (76) becomes

$$\frac{110 \times \cdot 0001 \times 4 \times 5280}{\Omega} + \frac{749}{\Omega} - \frac{309}{\Omega} = \cdot 16299\ F_2$$

which may be written as follows:—

$$\frac{672}{\Omega} = \cdot 16299\ F_2;\ \text{ or}$$

$$F_2 = \frac{4123}{\Omega}$$

where F_2 depends on the values of $\frac{\omega_1}{\Omega}$ and $\frac{\omega_2}{\Omega}$. We have to select trial values of Ω until we find one to satisfy this equation.

First, try the value $\Omega = 800$. Then we have

$\frac{\omega_1}{\Omega} = \cdot 93625$; $\frac{\omega_2}{\Omega} = \cdot 38625$; and from Table XXI the value of F_2 is found as follows:—

Corresponding with the value $\frac{\omega_2}{\Omega} = \cdot 40$, we have when $\frac{\omega_1}{\Omega}$ is $\cdot 93$, F_2 is 5 6479; and when $\frac{\omega_1}{\Omega}$ is $\cdot 95$, F_2 is 6·3625; hence, by proportional parts, when $\frac{\omega_1}{\Omega}$ is $\cdot 936$, F_2 is 5·8623. Again, corresponding with the value $\frac{\omega_2}{\Omega} = \cdot 35$, we have when $\frac{\omega_1}{\Omega}$ is $\cdot 93$, F_2 is 5·9646; and when $\frac{\omega_1}{\Omega}$ is $\cdot 95$, F_2 is 6·6792; hence by proportional parts when $\frac{\omega_1}{\Omega}$ is $\cdot 936$, F_2 is 6·1790.

We now have, for the value $\frac{\omega_1}{\Omega} = \cdot 936$, F_2 is 5·8623 when $\frac{\omega_2}{\Omega}$ is $\cdot 40$ and 6·1790 when $\frac{\omega_2}{\Omega}$ is $\cdot 35$.

Hence by proportional parts, for the value $\frac{\omega_2}{\Omega} = \cdot 386$ we have $F_2 = 5 \cdot 9510$.

The trial equation now becomes

$$F_2 = 5 \cdot 9510 = \frac{4123}{800} = 5 \cdot 1538.$$

These two values are inconsistent, so a fresh trial must be made

Next try $\Omega = 1000$;

Then we have $\frac{\omega_1}{\Omega} = \cdot 749$ and $\frac{\omega_2}{\Omega} = \cdot 309$ or very nearly $\cdot 75$ and $\cdot 31$ respectively.

The value of F_2 from Table XXI is thus 3·2648 and the equation becomes—

$$F_2 = 3 \cdot 2648 = \frac{4123}{1000} = 4 \cdot 123$$

This is also inconsistent.

Next try $\frac{\omega_1}{\Omega} = \cdot 85$ so that—

$$\Omega = \frac{749}{\cdot 85} = 881 \text{ and } \frac{\omega_2}{\Omega} = \frac{309}{749} \times \cdot 85 = \cdot 350$$

now $F_2 = 4 \cdot 2677$

and the equation is $4 \cdot 2677 = \frac{4123}{881} = 4 \cdot 680$

Again try $\frac{\omega_1}{\Omega} = \cdot 80$; $\frac{\omega_2}{\Omega} = \cdot 33$

Then $\Omega = \frac{749}{\cdot 8} = 932$ and $F_2 = 3 \cdot 703$

and the equation is $3 \cdot 703 = \frac{4123}{932} = 4 \cdot 4237$.

Again try $\frac{\omega_1}{\Omega} = \cdot 86$; $\frac{\omega_2}{\Omega} = \cdot 355$

$F_2 = 4 \cdot 4200$; $= \frac{749}{\cdot 86} = 871$

The equation is $4{\cdot}4200 = \frac{4123}{871} = 4{\cdot}74$

Again try $\frac{\omega_1}{\Omega} = {\cdot}90$; $\frac{\omega_2}{\Omega} = {\cdot}372$

$F_2 = 5{\cdot}0484$; $\Omega = \frac{749}{{\cdot}90} = 832$

The equation is $5{\cdot}0484 = \frac{4123}{832} = 4{\cdot}9555$

Again try $\frac{\omega_1}{\Omega} = {\cdot}89$; $\frac{\omega}{\Omega} = {\cdot}367$

Then $F_2 = 4{\cdot}8962$; $\Omega = \frac{749}{{\cdot}89} = 842$

The equation is $4{\cdot}8962 = \frac{4123}{842} = 4{\cdot}8991$.

This is very nearly exact.

The above results may be tabulated as follows:—

Ω	F_2	$\frac{4123}{\Omega}$
932	3·703	4·4237
881	4·2677	4·6800
871	4·4200	4·7400
842	4·8962	4·8991
832	5·0484	4·9555

It is seen that the correct value is $\Omega = 842$.

The value of Ω being 842 square feet, the depth of the canal is 7·81 feet. In calculating a value for ζ we assumed the depth to be 8 feet; and this small difference will not affect the result, so far as the value of ζ is concerned; no correction is therefore required.

We can now find the value of q by reference to Jackson's Tables. Its value is that corresponding with a depth of 7·81 feet and a bed slope of ·1 per 1,000. The value is, using proportional parts, 1,885 c.f.s.

Example 2:—

A canal has a bed-width of 50 feet, side slopes of 1 to 1, a longitudinal slope of ·2 per 1,000, and is discharging 1,370 c.f.s. At a certain place a fall in the bed occurs, which passes the discharge with a depth of 2 feet. What will be the depth at a place 1,000 feet upstream of the fall?

By reference to Jackson's Tables (page 283), we find that the "normal" depth of the canal, when the surface is parallel to the bed, is 8 feet, and the value of C is ·826. Hence $\zeta = \frac{64{\cdot}4}{(82{\cdot}6)^2}$ and $\frac{2i}{\zeta} = \frac{2 \times {\cdot}0002}{64{\cdot}4} \times (82{\cdot}6)^2 = {\cdot}042378$.

Next, the value of $\frac{b}{p}$ at a depth of 2 feet is $\frac{54}{55{\cdot}656} = {\cdot}97024$ and at a depth of 8 feet it is $\frac{66}{72{\cdot}624} = {\cdot}90878$. The factor $\left(1 - \frac{2i}{\zeta}\frac{b}{p}\right)$ thus lies somewhere between ·95888 and ·96149. It will be near enough to take it as ·960.

The remaining quantities are as follows:—

$\Omega = 464$; b (say) 60; $\omega_2 = 104$

$\frac{biL}{\Omega} = \frac{60 \times {\cdot}0002 \times 1000}{464} = \frac{12}{464} = {\cdot}025862$

$\frac{\omega_2}{\Omega} = \frac{104}{464} = {\cdot}22414$

And the equation is—

$025862 + \frac{\omega_1}{\Omega} - {\cdot}22414 = \frac{{\cdot}960}{6} F_2$

Which may be written $\frac{\omega_1}{\Omega} - {\cdot}16\, F_2 = {\cdot}1983$

1 First try $\frac{\omega_1}{\Omega} = \cdot 40$. Then, for the value $\frac{\omega_2}{\Omega} = \cdot 224$, we have from Table XXI, $F_2 = 1\cdot 091$
and the left hand side of the equation becomes $= \cdot 40 - \cdot 16 \times 1\cdot 091 = \cdot 225$

2 Next try $\frac{\omega_1}{\Omega} = \cdot 35$. Then $F_2 = \cdot 7746$
and we have $\cdot 35 - \cdot 16 \times \cdot 7746 = \cdot 226$

3 Next try $\frac{\omega_1}{\Omega} = \cdot 50$. Then $F_2 = 1\cdot 7462$
and $\cdot 50 - \cdot 16 \times 1\cdot 746 = \cdot 2206$

4 Next try $\frac{\omega_1}{\Omega} = \cdot 80$. Then $F_2 = 4\cdot 3485$
and $\cdot 80 - \cdot 16 \times 4\cdot 3485 = \cdot 10424$

5 Next try $\frac{\omega_1}{\Omega} = \cdot 60$. Then $F_2 = 2\cdot 4671$
and $\cdot 60 - \cdot 16 \times 2\cdot 4671 = \cdot 2053$

6 Next try $\frac{\omega_1}{\Omega} = \cdot 65$. Then $F_2 = 2\cdot 8656$
and $\cdot 65 - \cdot 16 \times 2\cdot 8656 = \cdot 1915$

By proportional parts between the values of $\frac{\omega_1}{\Omega}$ ·60 and ·65 we have, when the result is ·1983, then

$\frac{\omega_1}{\Omega} = \cdot 625$ and $F_2 = 2\cdot 6663$.

From this value of $\frac{\omega_1}{\Omega}$ we have $\omega_1 = \cdot 625 \times 464 = 290$ square feet, and the depth is 5·25 feet. This is the answer required.

From a practical point of view it should be noticed that, unless this canal is excavated in rock, the design is unsuitable, for the velocity at the fall will be $\frac{1370}{104} = 13\cdot 67$ feet per second; and at 1,000 feet from the fall it will be $\frac{1370}{290} = 4\cdot 73$ feet per second; velocities far too high for an earthen canal.

Example 3:—

It will often be found convenient to plot the whole section of the stream, by selecting successive convenient values of $\frac{\omega_1}{\Omega}$ and calculating the resulting values of L. Any desired intermediate values of $\frac{\omega_1}{\Omega}$ can be measured by scale from the surface-curve thus determined.

Suppose the bed-width to be 100 feet, side-slopes 1 to 1, longitudinal bed-slope ·1 per 1,000, and discharge 1,885 c.f.s. Suppose at the tail the water is backed up to a depth of 14·7 feet by a weir. As in example 1, we find from Jackson's Tables that the depth corresponding with a discharge of 1,885 c.f.s. is 7·81 feet, implying a cross sectional area of $\Omega = 842$ square feet. The value of ω_2 is $(114\cdot 7 \times 14\cdot 7 =)$ 1686, and the value of $\frac{\omega_2}{\Omega}$ is almost exactly 2·00.

The mean value of b is $\left(\frac{115\cdot 6 + 129\cdot 4}{2} =\right)$ 122·5 and the mean value of p is $\left(\frac{122+142}{2} =\right)$ 132, so that the mean value of $\frac{b}{p}$ is $\frac{122\cdot 5}{132} = \cdot 92804$.

The value of $\frac{2i}{\xi}$ is the same as in example 1, viz., ·022863; and the value of the factor $\frac{1}{6}\left(1 - \frac{2i}{\xi}\frac{b}{p}\right)$ is thus $\frac{1}{6}$ (1 — ·022863 × ·92804), that is, ·163986. Equation (73) now becomes—

$$\frac{122\cdot 5 \times \cdot 0001}{842} L = 2\cdot 00 - \frac{\omega_1}{\Omega} + \cdot 164 F_1.$$

That is, $L = 68734 \left(2\cdot 00 - \frac{\omega_1}{\Omega} + \cdot 164 F_1\right)$

The values of F, obtained by reference to Table XX, are as follows:—

$\frac{\omega_1}{\Omega} =$	1·01	1·05	1·10	1·20	1·30	1·40	1·50	1·60	1·70	1·80	1·90
$\frac{\omega_2}{\Omega} =$	2·00	2·00	2·00	2·00	2·00	2·00	2·00	2·00	2·00	2·00	2·00
$F_1 =$	7·725	4·584	3·203	2·088	1·448	1·033	·738	·517	·345	207	·094

and by calculation the following values of L are obtained—

$\cdot 164\ F_1 =$	1·2669	·7518	·5290	·3121	·2375	·1694	·1210	·0849	·0566	·0339	·0154
$L =$	155,124	116,972	98,220	78,522	61,438	52,683	42,681	33,332	21,510	16,077	7,932

TABLE XX.

Table showing values of the factor F_1,
where $F_1 = [\{L_1 + T_1\} - \{L_2 + T_2\}]$

and $L_1 = \log_e \left\{ \dfrac{\left(\frac{\omega_1}{\Omega}\right)^3 - 1}{\left(\frac{\omega_1}{\Omega} - 1\right)^3} \right\}$; $L_2 = \log_e \left\{ \dfrac{\left(\frac{\omega_2}{\Omega}\right)^3 - 1}{\left(\frac{\omega_2}{\Omega} - 1\right)^3} \right\}$

$T_1 = 3{\cdot}464 \tan^{-1}\left({\cdot}57736 + 1{\cdot}15472 \frac{\omega_1}{\Omega}\right)$; $T_2 = 3{\cdot}464 \tan^{-1}\left({\cdot}57736 + 1{\cdot}15472 \frac{\omega_2}{\Omega}\right)$

Applicable to a s'ream whose depth is *incre?sing* in the direction of flow, and consequently where $\omega_2 > \omega_1 > \Omega$ and $\frac{\omega_2}{\Omega} > \frac{\omega_1}{\Omega} > 1$

$\omega_1/\Omega=$													
5·00	$\omega_2/\Omega=$	5·50	6·00	6·50	7·00	8·00	9·00	10·00					
	F_1 =	·0202	·0371	·0494	·0591	·0734	·0834	·0903					
4·50	$\omega_2/\Omega=$	4·60	4·80	5·00	5·50	6·00	6·50	7·00	8·00	9·00	10·00		5·20
	F_1 =	·0066	·0133	·0255	·0487	·0656	·0779	·0876	·1019	·1119	·1183		·0378
4·00	$\omega_2/\Omega=$	4·20	4·40	4·60	4·80	5·00	5·50	6·00	6·50	7·00	8·00	9·00	10·00
	F_1 =	·0177	·0329	·0464	·0581	·0683	·0885	·1034	·1177	·1274	·1417	·1517	·1556
3·80	$\omega_2/\Omega=$	4·00	4·20	4·40	4·60	4·80	5·00	5·50	6·00	6·50	7·00	8·00	10·00
	F_1 =	·0196	·0373	·0525	·0660	·0777	·0879	·1081	·1230	·1373	·1470	·1613	·1782
3·60	$\omega_2/\Omega=$	3·80	4·00	4·20	4·50	4·80	5·00	5·20	5·50	6·00	7·00	8·00	10·00
	F_1 =	·0254	·0450	·0627	·0914	·1031	·1183	·1323	·1335	·1504	·1724	·1867	·2036
3·40	$\omega_2/\Omega=$	3·60	3·80	4·00	4·20	4·60	4·80	5·00	5·20	5·50	6·00	8·00	10·00
	F_1 =	·0274	·0537	·0783	·0918	·1107	·1314	·1416	·1506	·1618	·1787	·2150	·2319
3·20	$\omega_2/\Omega=$	3·40	3·60	3·80	4·00	4·20	4·40	·460	5·00	5·50	6·00	8·00	10·00
	F_1 =	·0346	·0620	·0885	·1070	·1236	·1408	·1543	·1762	·1964	·2153	·2196	·2685
3·00	$\omega_2/\Omega=$	3·20	3·40	3·60	3·80	4·00	4·20	4·50	5·00	5·50	6·00	8·00	10·00
	F_1 =	·0423	·0763	·1037	·1300	·1496	·1673	·1894	·2179	·2391	·2550	·2913	·3082
2·90	$\omega_2/\Omega=$	3·00	3·10	3·20	3·40	3·60	3·80	4·00	4·50	5·00	6·00	8·00	10·00
	F_1 =	·0244	·0463	·0661	·1007	·1281	·1544	·1740	·2133	·2423	·3794	·3157	·3326
2·80	$\omega_2/\Omega=$	2·90	3·00	3·10	3·20	3·40	3·60	3·80	4·00	5·00	6·00	8·00	10·00
	F_1 =	·0270	·0514	·0733	·0931	·1277	·1551	·1514	·2010	·2693	·3061	·3427	·3391
2·70	$\omega_2/\Omega=$	2·80	2·90	3·00	3·10	3·20	3·40	3·60	4·00	5·00	6·00	8·00	10·00
	F_1 =	·0306	·0576	·0819	·1038	·1236	·1582	·1865	·2315	·2908	·3369	·3732	·3301
2·60	$\omega_2/\Omega=$	2·70	2·80	2·90	3·00	3·10	3·20	3·50	4·00	5·00	6·00	8·00	10·00
	F_1 =	·0340	·0645	·0915	·1159	·1375	·1576	·2070	·2655	·3338	·3709	·4072	·4241
2·50	$\omega_2/\Omega=$	2·60	2·70	2·80	2·90	3·00	3·20	3·50	4·00	5·00	6·00	8·00	10·00
	F_1 =	·0387	·0727	·1033	·1302	·1546	·1963	·2457	·3043	·3725	·4090	·4459	·4623
2·40	$\omega_2/\Omega=$	2·50	2·60	2·70	2·80	2·90	3·00	3·20	3·50	4·00	5·00	6·00	8·00
	F_1 =	·0433	·0825	·1165	·1470	·1740	·1984	·2401	·2895	·3180	·4103	·4534	·4597
2·30	$\omega_2/\Omega=$	2·40	2·50	2·60	2·70	2·80	3·00	3·20	3·50	4·00	5·00	6·00	8·00
	F_1 =	·0501	·0939	·1326	·1680	·1971	·2485	·2902	·3306	·3961	·4664	·5035	·3393
2·20	$\omega_2/\Omega=$	2·30	2·40	2·50	2·60	2·70	2·80	3·00	3·20	3·50	4·00	5·00	6·00
	F_1 =	·0578	·1079	·1517	·1904	·2241	·2549	·3063	·3480	·3974	·4559	·5242	·5613
2·10	$\omega_2/\Omega=$	2·20	2·30	2·40	2·50	2·60	2·80	3·00	3·20	3·50	4·00	5·00	6·00
	F_1 =	·0673	·1251	·1751	·2190	·2377	·3233	·3756	·4153	·4517	·5332	·5915	·6256
2·00	$\omega_2/\Omega=$	2·05	2·10	2·15	2·20	2·30	2·40	2·50	2·60	2·80	3·00	3·50	4·00
	F_1 =	·0410	·0787	·1130	·1460	·2038	·2539	·2977	·3364	·4009	·4523	·5184	·6019
1·95	$\omega_2/\Omega=$	2·00	2·0	2·10	2·15	2·20	2·30	2·40	2·60	2·80	3·00	3·50	4·00
	F_1 =	·0448	·0858	·1235	·1581	·1908	·2435	·2957	·3312	·4437	·4971	·5582	·647
1·90	$\omega_2/\Omega=$	1·95	2·00	2·05	2·10	2·20	2·30	2·40	2·60	2·80	3·00	3·50	4·00
	F_1 =	·0480	·093	·1347	·1724	·2307	·2975	·3476	·4301	·4946	·5460	·6371	·6952
1·85	$\omega_2/\Omega=$	1·90	1·95	2·00	2·05	2·10	2·20	2·30	2·40	2·60	2·80	3·00	3·50
	F_1 =	·0537	·1026	·1474	·1864	·2201	·2934	·3512	·4013	·4538	·5483	·5997	·6908

	5·20												
	·0773												
	5·20												
	·0939												
10·00													
·6660													
10·00													
·6507													
8·00	10·00												
·6576	·6115												
8·00	10·00												
·6649	·6518												
6·00	0·00	8·00	10·00										
·6702	·7073	·7456	·7606										
6·00	6·00	8·00	10·00										
·7150	·7621	·7684	·8053										
6·00	6·00	8·00	10·00										
·7611	·8010	·8378	·8642										
4·00	6·00	0·00	8·00	10·00									
·7493	·8176	·8647	·8910	·9070									

$\omega/\Omega=$													
1·80	$\omega_2/\Omega=$	1·85	1·90	1·95	2·00	2·10	2·20	2·30	2·40	2·50	2·60	3·00	3·50
	F_1 =	·0593	·1130	·1621	·2067	·2634	·3327	·4103	·4606	·5431	·6070	·6590	·7501
1·75	$\omega_2/\Omega=$	1·80	1·85	1·90	1·95	2·00	2·10	2·20	2·30	2·40	2·50	2·60	3·00
	F_1 =	·0652	·1245	·1782	·2271	·2710	·3500	·4179	·4767	·5258	·6083	·6728	·7242
1·70	$\omega_2/\Omega=$	1·75	1·80	1·85	1·90	1·95	2·00	2·10	2·20	2·30	2·40	2·60	2·80
	F_1 =	·0729	·1391	·1974	·2511	·3000	·3419	·4235	·4909	·5186	·5967	·6612	·7457
1·65	$\omega_2/\Omega=$	1·70	1·75	1·80	1·85	1·90	1·95	2·00	2·10	2·20	2·30	2·40	2·60
	F_1 =	·0810	·1539	·2191	·2784	·3321	·3310	·4258	·5045	·5718	·6256	·6707	·7623
1·60	$\omega_2/\Omega=$	1·62	1·64	1·66	1·68	1·70	1·75	1·80	1·85	1·90	2·00	2·20	2·50
	F_1 =	·0350	·0741	·1052	·1413	·1722	·2431	·3103	·3606	·4233	·5170	·6630	·8147
1·58	$\omega_2/\Omega=$	1·60	1·62	1·64	1·66	1·68	1·70	1·75	1·80	1·90	2·00	2·20	2·50
	F_1 =	·0399	·0770	·1140	·1491	·1812	·2121	·2850	·3502	·4632	·5569	·7020	·8546
1·56	$\omega_2/\Omega=$	1·58	1·60	1·62	1·64	1·66	1·70	1·75	1·80	1·90	2·00	2·20	2·50
	F_1 =	·0417	·0816	·1106	·1137	·1599	·2338	·3267	·3919	·5049	·5956	·7446	·8963
1·54	$\omega_2/\Omega=$	1·56	1·58	1·60	1·62	1·64	1·66	1·70	1·80	1·90	2·00	2·20	2·50
	F_1 =	·0443	·0860	·1250	·1630	·2000	·2311	·2991	·4362	·5492	·6431	·7889	·9406
1·52	$\omega_2/\Omega=$	1·54	1·56	1·58	1·60	1·62	1·66	1·70	1·80	1·90	2·00	2·20	2·50
	F_1 =	·0464	·0907	·1324	·1733	·2103	·2635	·3445	·4936	·5956	·6893	·8353	·9970
1·50	$\omega_2/\Omega=$	1·52	1·54	1·56	1·58	1·60	1·65	1·70	1·80	1·90	2·00	2·20	2·50
	F_1 =	·0490	·0954	·1307	·1914	·2213	·3125	·3935	·5316	·6446	·7353	·8913	1·0360
1·48	$\omega_2/\Omega=$	1·49	1·50	1·52	1·55	1·60	1·65	1·70	1·80	1·90	2·00	2·20	2·50
	F_1 =	·0263	·0521	·1011	·1701	·2731	·3646	·4456	·5837	·6967	·7904	·9364	1·0991
1·46	$\omega_2/\Omega=$	1·47	1·48	1·50	1·55	1·60	1·65	1·70	1·80	1·90	2·00	2·20	2·50
	F_1 =	·0275	·0545	·1069	·2249	·3252	·4104	·5001	·6393	·7515	·8442	·9312	1·1429
1·44	$\omega_2/\Omega=$	1·45	1·47	1·50	1·55	1·60	1·65	1·70	1·80	1·90	2·00	2·20	2·50
	F_1 =	·0297	·0864	·1633	·2533	·3571	·4753	·5503	·6974	·8104	·9011	1·0501	1·2018
1·42	$\omega_2/\Omega=$	1·43	1·45	1·47	1·50	1·55	1·60	1·70	1·80	1·90	2·00	2·20	2·50
	F_1 =	·0319	·0921	·1455	·2282	·3402	·4405	·6217	·7599	·8723	·9665	1·1125	1·2542
1·40	$\omega_2/\Omega=$	1·41	1·42	1·43	1·45	1·50	1·55	1·60	1·70	1·80	1·90	2·00	2·20
	F_1 =	·0341	·0667	·0986	·1588	·2949	·4129	·5162	·6864	·8265	·9385	1·0332	1·1709
1·38	$\omega_2/\Omega=$	1·39	1·40	1·41	1·42	1·45	1·50	1·55	1·60	1·70	1·80	1·90	2·00
	F_1 =	·0360	·0709	·1050	·1376	·2207	·3658	·4539	·5571	·7593	·8974	1·0104	1·1041
1·36	$\omega_2/\Omega=$	1·37	1·38	1·39	1·40	1·42	1·45	1·50	1·55	1·60	1·70	1·80	1·90
	F_1 =	·0396	·0764	·1124	·1473	·2140	·3001	·4422	·5602	·6635	·8357	·9738	1·0863
1·34	$\omega_2/\Omega=$	1·35	1·36	1·37	1·38	1·40	1·45	1·50	1·55	1·60	1·70	1·80	1·90
	F_1 =	·0418	·0824	·1210	·1553	·2207	·3553	·5246	·6426	·7430	·8181	1·0558	1·1692
1·32	$\omega_2/\Omega=$	1·33	1·34	1·35	1·37	1·40	1·45	1·50	1·55	1·60	1·70	1·80	1·90
	F_1 =	·0456	·0896	·1304	·2000	·3153	·4771	·6132	·7312	·8345	1·0067	1·1448	1·2578
1·30	$\omega_2/\Omega=$	1·31	1·32	1·33	1·35	1·37	1·40	1·45	1·50	1·55	1·60	1·70	1·80
	F_1 =	·0486	·0901	·1417	·2365	·3057	·4144	·5733	·7093	·8278	·9208	1·1029	1·2409
1·28	$\omega_2/\Omega=$	1·29	1·30	1·31	1·33	1·35	1·37	1·40	1·45	1·50	1·55	1·60	1·70
	F_1 =	·0530	·1050	·1536	·2407	·3315	·4107	·5194	·6783	·8143	·9333	1·0356	1·2678

4·00	5·00	0·00	8·00	10·00									
·8080	·8771	·9140	·9503	·9672									
3·50	4·00	5·00	6·00	8·00	10·00								
·8163	·8739	·9121	·9702	1·0155	1·0324								
3·00	3·50	4·00	5·00	6·00	8·00	1·000							
·7071	·8852	·9467	1·0150	1·0521	1·0881	1·1033							
2·60	3·00	3·50	4·00	5·00	0·00	8·00	10·00						
·8207	·8781	·9693	1·0277	1·0960	1·1331	1·1694	1·1863						
3·00	4·00	5·00	6·00	0·00	10·00								
·9693	1·1180	1·1872	1·2249	1·2600	1·2775								
3·00	4·00	5·00	0·00	8·00	10·00								
1·0092	1·1553	1·2271	1·2612	1·3005	1·3174								
3·00	4·00	5·00	0·00	0·00	10·00								
1·0509	1·2005	1·2689	1·3038	1·3422	1·3560								
3·00	4·00	5·00	6·00	0·00	10·00								
1·0952	1·2449	1·3151	1·3502	1·3865	1·4034								
3·00	4·00	5·00	0·00	8·00	10·00								
1·1410	1·2912	1·3505	1·3960	1·4320	1·4498								
3·00	4·00	5·00	0·00	0·00	10·00								
1·1966	1·3402	1·4065	1·4450	1·4519	1·4988								
3·00	4·00	5·00	6·00	0·00	10·00								
1·2427	1·3923	1·4605	1·4977	1·5310	1·5509								
3·00	4·00	5·00	6·00	0·00	10·00								
1·2975	1·4471	1·5154	1·5525	1·5868	1·6037								
3·00	4·00	5·00	0·00	0·00	10·00								
1·3561	1·5060	1·5743	1·6114	1·6477	1·6646								
3·00	4·00	5·00	0·00	8·00	10·00								
1·4185	1·5684	1·6307	1·6733	1·7101	1·7270								
2·50	3·00	4·00	5·00	0·00	0·00	10·00							
1·3309	1·4655	1·6351	1·7031	1·7405	1·7768	1·7937							
2·20	2·50	3·00	4·00	5·00	0·00	0·00	10·00						
1·2501	1·4019	1·5504	1·7060	1·7743	1·8114	1·8477	1·8646						
2·00	2·20	2·50	3·00	4·00	5·00	0·00	0·00	10·00					
1·1905	1·3205	1·4782	1·6328	1·7524	1·8207	1·8578	1·8241	1·9111					
2·00	2·20	2·50	3·00	4·00	5·00	0·00	8·00	10·00					
1·2020	1·4050	1·5590	1·7152	1·8649	1·9331	1·9703	2·0065	2·0234					
2·00	2·20	2·50	3·00	4·00	5·00	0·00	8·00	10·00					
1·2516	1·4075	1·6402	1·8038	1·9034	2·0217	2·0593	2·0951	2·1120					
1·00	2·00	2·20	2·50	3·00	4·00	5·00	0·00	0·00	10·00				
1·2539	1·4476	1·6030	1·7165	1·8909	2·0405	2·1178	2·1519	2·1912	2·2051				
1·00	1·90	2·00	2·20	2·50	3·00	4·00	6·00	0·00	0·00	10·00			
1·2404	1·4559	1·6220	1·7086	1·8803	2·0049	2·1545	2·2228	2·2599	2·2962	2·3131			

$\omega_1/\Omega=$													
1·26	$\omega_2/\Omega=$	1·27	1·28	1·29	1·30	1·32	1·35	1·40	1·45	1·50	1·55	1·60	1·70
	F_1 =	·0590	·1145	·1681	·2105	·3156	·4460	·6341	·7027	·9233	1·0472	1·1601	1·3237
1·24	$\omega_2/\Omega=$	1·25	1·26	1·27	1·28	1·30	1·32	1·35	1·40	1·45	1·50	1·55	1·60
	F_1 =	·0647	·1261	·1851	·2100	·3156	·4117	·5721	·7601	·9168	1·0549	1·1729	1·2702
1·22	$\omega_2/\Omega=$	1·23	1·24	1·25	1·26	1·28	1·30	1·32	1·35	1·40	1·45	1·50	1·60
	F_1 =	·0716	·1392	·2039	·2653	·3708	·4358	·5909	·7113	·8992	1·0530	1·1941	1·4154
1·20	$\omega_2/\Omega=$	1·21	1·22	1·23	1·24	1·26	1·28	1·30	1·32	1·35	1·40	1·45	1·50
	F_1 =	·0798	·1555	·2270	·2947	·4208	·5353	·6403	·7364	·8669	1·0547	1·2135	1·3400
1·18	$\omega_2/\Omega=$	1·19	1·20	1·21	1·22	1·24	1·26	1·28	1·30	1·35	1·40	1·45	1·50
	F_1 =	·0905	·1755	·2553	·3310	·4702	·5903	·7108	·8158	1·0123	1·2302	1·3390	1·5251
1·16	$\omega_2/\Omega=$	1·17	1·18	1·19	1·20	1·22	1·24	1·27	1·30	1·35	1·40	1·45	1·50
	F_1 =	·1033	·2030	·2905	·3755	·5310	·6702	·8553	1·0153	1·2423	1·4302	1·5390	1·7251
1·14	$\omega_2/\Omega=$	1·15	1·16	1·17	1·18	1·19	1·20	1·22	1·24	1·27	1·30	1·35	1·40
	F_1 =	·1196	·2305	·3338	·4303	·5210	·6060	·7615	·9007	1·0868	1·2163	1·4728	1·6607
1·12	$\omega_2/\Omega=$	1·13	1·14	1·15	1·16	1·17	1·18	1·19	1·20	1·22	1·24	1·27	1·30
	F_1 =	·1416	·2717	·3913	·5022	·6035	·7022	·7987	·8777	1·0332	1·1724	1·3576	1·5187
1·10	$\omega_2/\Omega=$	1·11	1·12	1·13	1·14	1·15	1·16	1·18	1·20	1·22	1·24	1·27	1·30
	F_1 =	·1720	·3273	·4733	·5970	·7156	·8205	1·0293	1·2050	1·3605	1·4997	1·6843	1·8453
1·09	$\omega_2/\Omega=$	1·10	1·11	1·12	1·13	1·14	1·15	1·16	1·18	1·20	1·22	1·24	1·27
	F_1 =	·1919	·3639	·5192	·6607	·7909	·9105	1·0214	1·2214	1·3960	1·5521	1·6916	1·8767
1·08	$\omega_2/\Omega=$	1·09	1·10	1·11	1·12	1·13	1·14	1·15	1·16	1·18	1·20	1·22	1·25
	F_1 =	·3167	·4056	·5936	·7350	·8774	1·0076	1·1272	1·2351	1·4351	1·6136	1·7691	1·9730
1·07	$\omega_2/\Omega=$	1·08	1·09	1·10	1·11	1·12	1·13	1·14	1·16	1·18	1·20	1·22	1·25
	F_1 =	·2483	·4650	·6569	·8259	·9812	1·1257	1·2559	1·4864	1·6564	1·8619	2·0174	2·2213
1·06	$\omega_2/\Omega=$	1·07	1·08	1·09	1·10	1·11	1·12	1·13	1·14	1·16	1·18	1·20	1·22
	F_1 =	·2890	·5373	·7510	·9459	1·1179	1·2733	1·4147	1·5410	1·7754	1·9754	2·1509	2·3064
1·05	$\omega_2/\Omega=$	1·06	1·07	1·08	1·09	1·10	1·11	1·12	1·14	1·16	1·18	1·20	1·22
	F_1 =	·3155	·6345	·8928	1·0985	1·2914	1·4734	1·6157	1·8904	2·1209	2·3209	2·4964	2·6519
1·04	$\omega_2/\Omega=$	1·05	1·06	1·07	1·08	1·09	1·10	1·11	1·12	1·14	1·16	1·18	1·20
	F_1 =	·4265	·7720	1·0010	1·3003	1·5260	1·7179	1·8590	2·0453	2·3160	2·5474	2·7474	2·9229
1·03	$\omega_2/\Omega=$	1·04	1·05	1·06	1·07	1·08	1·09	1·10	1·12	1·14	1·16	1·18	1·20
	F_1 =	·5561	·9926	1·3331	1·6171	1·8654	2·0521	2·2740	2·5013	2·8730	3·1055	3·3035	3·4790
1·02	$\omega_2/\Omega=$	1·03	1·04	1·05	1·06	1·07	1·08	1·09	1·10	1·12	1·14	1·16	1·18
	F_1 =	·7011	1·3472	1·7737	2·1192	2·4093	2·6565	2·8732	3·0651	3·3021	3·6641	3·8946	4·0945
1·01	$\omega_2/\Omega=$	1·02	1·03	1·04	1·05	1·06	1·07	1·08	1·09	1·10	1·12	1·14	1·16
	F_1 =	1·3668	2·1579	2·7140	3·1405	3·4200	3·7750	4·0233	4·2100	4·4319	4·7593	5·0300	5·2614

1·60	1·00	2·00	2·20	2·50	3·00	4·00	5·00	6·00	8·00	10·00				
1·1001	1·5754	1·6071	1·8131	1·0649	2·1104	2·2600	2·3373	2·3744	2·4107	2·4276				
1·70	1·80	2·00	2·20	2·50	3·00	4·00	5·00	6·00	8·00	10·00				
1·1491	1·5865	1·7032	1·0302	2·0000	2·2453	2·3031	2·4631	2·5065	2·5368	2·5537				
1·70	1·80	2·00	2·20	2·50	3·00	4·00	5·00	0·00	8·00	10·00				
1·0376	1·7257	1·0324	2·7784	2·2301	2·3347	2·5343	2·6026	2·6307	2·6760	2·6020				
1·00	1·70	1·80	2·00	2·20	2·50	3·00	4·00	5·00	0·00	8·00	10·00			
1·5709	1·7431	1·8812	2·0891	2·2330	2·3556	2·5102	2·6808	2·7581	2·7052	2·8315	2·8481			
1·60	1·70	1·00	2·00	2·20	2·50	3·00	4·00	5·00	0·00	8·00	10·00			
1·7464	1·9180	2·0567	2·2634	2·4094	2·5611	2·7157	2·8033	2·0330	2·0707	3·0070	3·0230			
1·00	1·70	1·80	2·00	2·20	2·50	3·00	4·00	5·00	0·00	0·00	10·00			
1·0164	2·3186	2·2507	2·4634	2·6004	2·7611	2·0157	3·0053	3·1336	3·1707	3·2070	3·2230			
1·45	1·50	1·60	1·70	1·80	2·00	2·20	2·50	3·00	4·00	5·00	8·00	8·00	10·00	
1·8105	1·0556	2·1769	2·3191	2·4872	2·6930	2·8300	2·9016	3·1162	3·2959	3·3611	3·4013	3·4373	3·4611	
1·35	1·40	1·45	1·50	1·00	1·70	1·80	1·90	2·00	2·20	2·50	3·00	4·00		5·00
1·7415	1·0324	2·0012	2·2273	2·4180	2·6209	2·7589	2·8710	2·9650	3·1116	3·2733	3·1170	3·9670		3·6055
1·35	1·40	1·45	1·50	1·00	1·70	1·00	1·90	2·00	2·20	2·50	3·00	4·00		5·00
2·0718	2·2507	2·4185	2·5546	2·7761	2·0161	3·0563	3·1002	3·2020	3·4359	3·5000	3·7452	3·6948		3·9031
1·30	1·35	1·40	1·45	1·50	1·00	1·70	1·80	1·00	2·00	2 20	2·50	3·00	4·00	5·00
2·0372	2·2037	2·4516	2·6104	2·7165	2·9676	3·1100	3·2781	3·3911	3·1949	3·6308	3·7625	3·0371	4·0567	4·1570
1·30	1·35	1·40	1·45	1·50	1·00	1·70	1·80	1·90	2·00	2 20	2·50	3·00	4·00	5·00
2·2530	2·4501	2·6083	2·8271	2·9632	3·1815	3·3507	3·4918	3·6078	3·7015	3·8475	3·0092	4·1559	4·3034	4·3717
1·20	1·35	1·40	1·45	1·50	1·00	1·70	1·80	1·00	2·00	2·20	2·50	3·00	4·00	5·00
2·6023	2·7297	2·0166	3·1764	3·2115	3·4338	3·6050	3·7431	3·8561	3·0198	4·0968	4·2475	4·4021	4·5517	4·6200
1·25	1·30	1·35	1·40	1·45	1·50	1·00	1·70	1·00	2·00	2·20	2·50	3·00	4·00	5·00
2·5103	2·7912	3·0177	3·2056	3·3614	3·5005	3·7218	3·6910	4·0321	4·2399	4·3918	4·5305	4·6911	4·8407	4·9099
1·25	1·30	1·35	1·40	1·45	1·50	1·00	1·70	1 00	2·00	2·20	2·50	3·00	4·00	5 00
2·8553	3·1307	3·3632	3·5511	3·7099	3·8160	4·0073	4·2393	4·3776	4·5913	1·7303	4 8520	5·0366	5·1862	5·2545
1·22	1·25	1·30	1·35	1·40	1·45	1·50	1·00	1·70	1·00	2·00	2·20	2·50	3·00	4·00
3·0784	3·2823	3·5033	3·7597	3·9770	4·1361	4·2725	4·1030	4·6600	4·8011	5·0108	5·1508	5·3665	5·4631	0·6127
1·22	1·25	1·30	1·35	1·40	1·45	1·50	1·00	1·70	1·80	2·00	2·20	2·50	3·00	4·00
3·6345	3·8384	4·1103	4·3168	4·5337	4·6926	4·8256	5·0109	5·2221	5·3602	5·5680	5·7129	0·8646	5·0193	6·1758
1·20	1·22	1·25	1·30	1·35	1·40	1·45	1·50	1·00	1·00	2·00	2·20	2·50	3·00	4·00
4·2701	4·4256	4·6295	4·0101	0·1369	5·3218	5·4930	5·6107	5·8110	6·1313	6·3580	6·5010	6·6557	6·5103	6·9599
1·10	1·20	1·22	1·25	1·30	1·35	1·40	1·50	1·00	1·80	2·00	2·20	2·50	3·00	4·00
5·4914	6·0369	5·7924	5·9963	6·2772	6·5037	6·6910	6·0565	7·2052	7·5181	7·7219	7·6709	8·0225	8·1771	8·3267

TABLE XXI.

Table showing values of the factor F_2,

where $F_2 = (L_1 + T_1) - (L_2 + T_2)$

and

$$L_1 = \log_e \left\{ \frac{1 - \left(\frac{\omega_1}{\Omega}\right)^3}{\left(1 - \frac{\omega_1}{\Omega}\right)^3} \right\}; \qquad L_2 = \log_e \left\{ \frac{1 - \left(\frac{\omega}{\Omega}\right)^3}{\left(1 - \frac{\omega_2}{\Omega}\right)^3} \right\}$$

$$T_1 = 3{\cdot}464 \tan^{-1}\left(\cdot 57736 + 1{\cdot}15472 \frac{\omega_1}{\Omega}\right); \quad T_2 = 3{\cdot}464 \tan^{-1}\left(\cdot 57736 + 1{\cdot}15472 \frac{\omega_2}{\Omega}\right)$$

Applicable to a stream whose depth is *decreasing in* the direction of flow, where $\Omega > \omega_1 > \omega_2$

$\omega_1/\Omega=$											
·10	$\omega_2/\Omega=$	·09	·08	·07	·06	·05	·04	·03	·02	·01	
	F_2 =	·0602	·1203	·1832	·2403	·3003	·3603	·4203	·4803	·5404	
·20	$\omega_2/\Omega=$	·19	·18	·17	·16	·15	·14	·13	·12	·11	·10
	F_2 =	·0605	·1209	·1813	·2414	·3021	·3619	·4220	·4821	·5422	·6021
·25	$\omega_2/\Omega=$	·24	·23	·22	·21	·20	·19	·18	·17	·16	·15
	F_2 =	·0610	·1217	·1824	·2430	·3035	·3640	·4244	·4848	·5449	·6050
·30	$\omega_2/\Omega=$	·28	·26	·27	·26	·25	·24	·23	·22	·21	·20
	F_2 =	·0615	·1230	·1842	·2454	·3064	·3674	·4281	·4888	·5494	·6099
·35	$\omega_2/\Omega=$	·34	·33	·32	·31	·30	·28	·26	·24	·22	·20
	F_2 =	·0626	·1250	·1870	·2490	·3107	·4337	·5561	·6781	·7995	·9206
·40	$\omega_2/\Omega=$	·39	·38	·37	·36	·35	·33	·31	·29	·27	·25
	F_2 =	·0640	·1277	·1910	·2540	·3169	·4412	·5638	·6891	·8118	·9310
·45	$\omega_2/\Omega=$	·44	·43	·42	·41	·40	·38	·36	·34	·32	·30
	F_2 =	·0659	·1312	·1962	·2608	·3250	·4527	·5790	·7045	·8289	·9526
·50	$\omega_2/\Omega=$	·49	·48	·47	·46	·44	·42	·40	·38	·35	·30
	F_2 =	·0683	·1360	·2033	·2699	·4021	·5324	·6612	·7850	·9781	1·2659
·55	$\omega_2/\Omega=$	·54	·53	·52	·51	·50	·48	·46	·43	·40	·35
	F_2 =	·0714	·1423	·2124	·2810	·3507	·4577	·6200	·8181	1·0110	1·3183
·60	$\omega_2/\Omega=$	·59	·58	·57	·56	·55	·52	·50	·45	·45	·35
	F_2 =	·0750	·1510	·2250	·2982	·3123	·5833	·7236	1·0578	1·3328	1·6997
·65	$\omega_2/\Omega=$	·64	·63	·62	·60	·58	·55	·50	·45	·40	·35
	F_2 =	·0820	·1629	·2421	·3074	·5494	·7083	1·1100	1·4532	1·7802	2·0971
·70	$\omega_2/\Omega=$	·68	·68	·67	·65	·63	·60	·55	·50	·45	·40
	F_2 =	·0902	·1788	·2580	·4338	·5960	·8312	1·2021	1·5523	1·8800	2·2140
·75	$\omega_2/\Omega=$	·74	·73	·72	70	·60	·65	·60	·55	·50	·45
	F_2 =	·1033	·2019	·2988	·4857	·6743	·9185	1·3160	1·6878	2·0355	2·3747
·80	$\omega_2/\Omega=$	·79	·78	·77	·75	·73	·70	·65	·60	·55	·50
	F_2 =	·1210	·2367	·3490	·5630	·7610	1·0457	1·4825	1·8799	2·2608	2·6015
·85	$\omega_2/\Omega=$	·84	·83	·82	·80	·78	·75	·70	·65	·60	·55
	F_2 =	·1513	·2910	·4318	·6863	·9250	1·2513	1·7370	2·1708	2·5693	2·9391
·90	$\omega_2/\Omega=$	·89	·88	·87	·85	·83	·80	·75	·70	·65	·60
	F_2 =	·2123	·4079	·5890	·9198	1·2144	1·6061	2·1711	2·6572	3·0906	3·1580
·93	$\omega_2/\Omega=$	·92	·91	·90	·88	·85	·80	·75	·70	·65	·60
	F_2 =	·2880	·5449	·7768	1·1847	1·6966	2·3819	2·9479	3·1536	3·8671	4·2048
·95	$\omega_2/\Omega=$	·94	·93	·92	·90	·88	·85	·80	·75	·70	·65
	F_2 =	·3854	·7146	1·0020	1·4914	1·8993	2·4112	3·0095	3·6925	4·1482	4·5820
·97	$\omega_2/\Omega=$	·96	·95	·94	·92	·90	·85	·80	·75	·70	·65
	F_2 =	·5062	1·0629	1·4483	2·0655	2·5543	3·1741	4·1621	4·7254	5·2111	5·6449
·98	$\omega_2/\Omega=$	·97	·96	·95	·94	·92	·90	·85	·80	·75	·70
	F_2 =	·8316	1·4377	1·8944	2·2708	2·8070	3·3858	4·3056	4·9959	5·5509	6·0426
·99	$\omega_2/\Omega=$	·98	·97	·96	·95	·93	·90	·85	·80	·75	·70
	F_2 =	1·4062	2·2377	2·8341	3·3006	4·0152	4·7920	5·7118	6·4001	6·9651	7·4158

·90	·05	·05									
·6023	·7823	·9024									
·14	·12	·09	·05								
·6654	·7856	·9038	1·2050								
·18	·16	·12	·09	·05							
·7308	·0120	1·0020	1·2722	1·5123							
·10	·15	·12	·00	·05							
1·0415	1·2227	1·4027	1·5899	1·8230							
·23	·20	·16	10	·05							
1·0557	1·2375	1·6300	1·8896	2·0390							
·25	·20	·16	·10	·05							
1·2590	1·5025	1·8040	2·1040	2·4619							
·25	·20	·16	·10	·05							
1·5952	1·8957	1·2008	1·5003	1·8011							
·30	·25	·20	·15	·10							
1·6395	1·0350	2·2494	2·5515	2·8515							
·30	·25	·20	·16	·10							
2·0101	2·3163	2·6203	2·0224	3·2224							
·30'	·25	·20	·15	·10							
2·4073	2·7142	3·0177	3·3108	3·6198							
·35	·30	·25	·20	·15	·10						
2·5309	2·8110	3·1490	3·4515	3·7536	4·0536						
·40	·35	·30	·25	·20	·15	·10					
3·0097	3·0166	3·3273	3·6337	3·9372	4·2303	4·5303					
·45	·40	·35	·30	·25	·20	·15	·10				
2·9377	3·2527	3·5700	3·8003	4·1907	4·5002	4·8023					
·50	·45	·40	·35	·30	·25	·20	·15	·10			
3·2808	3·6200	3·0310	4·2070	4·5730	4 8830	5·1855	5·4007				
·55	·50	·45	·40	·35	·30	·25	·20	·15	·10		
3·6559	4·2506	4·5158	4·8708	5·1177	5·4094	5·8048	0·1033	6·4104	6·7104		
·55	·50	·45	·40	·35	·30	·25	·20	·15	·10		
4·0351	4·0561	0·3227	5·0170	5·9645	6·2752	6 5910	6·8851	7·1573	7·4872		
·60	·55	·50	·45	·40	·35	·30	·25	·20	·15	·10	
4 0704	5·3503	5·7010	6·0372	6·3632	0·6701	6·1999	7·2902	7 5907	7·0018	8·2018	
·60	·55	·50	·45	·40	·35	30	·25	·20	·15	·10	
6·0123	6·4102	0·7030	7·1001	7·4251	7·7420	8·0527	8·3501	8·0020	8·0047	0·2047	
·65	·60	·60	·50	·45	·40	·35	·30	·25	·20	·15	·10
6·4704	6·6738	7·2147	7·6351	7·0310	8·2560	8·5735	8·8813	9·1000	9·4041	9·7952	10·0962
·65	·60	·55	·50	·45	·40	·85	·30	·25	·20	·15	·10
7·8920	8·2800	8·0500	9·0010	9·3378	9·0028	9·0797	10·2001	10·5068	10·9003	11·2024	11·6024

IX.—EXAMPLES OF DRAINAGE SCHEMES.

In my last lecture I propose to show you some examples of actual drainage-schemes which have come before me in the course of business, and which embody principles that have to be frequently used in making calculations for such schemes, but cannot be very conveniently expressed in a general form.

IX (a).—DRAINAGE OF THE ARGOWAL CIRCUIT.

The Argowal Circuit (see plan after page 74) is the name given to an embankment in the Midnapore district, which was originally constructed a very long time ago for the protection of certain lands subjected to floods from the Bagda river and the Sadar Khal, or Etabaria Khal, as well as from the floods which sweep over the country from certain rivers farther inland. The embankment is continuous, and forms a complete ring-bund round the interior lands, with the intention of completely excluding the flood-water. The effect has, however, been such as was certainly not anticipated by the people who made the embankment. The rivers here are very heavily charged with silt, and as the river-water has been completely excluded from the lands inside the circuit, these lands have received no deposit of silt, and consequently have remained at their original levels. Not so the lands outside the circuit, along the margins of the rivers. These lands (*jalpai* lands as they are called) have, day by day, and year by year, been receiving deposits of silt from the rivers, until they have, in process of time, been raised to a level very much higher than the lands inside the circuit. The natural process of land formation, which I have described before, has, in this case, been artificially prevented from taking place inside the embankment, and the interior lands have been left unraised. The evil effects of this are seen when high floods occur in the rivers. Before the embankment was made, the water used to spread over the country, without doing much harm. This cannot occur now, for a double reason; first, the existence of the embankment shuts out a great deal of waterway from the flood-spill, and thus throws a greater quantity of water into the river bed; and secondly, the capacity of the river bed itself is decreased largely by the silted-up high lands along the margins, and it can no longer carry off the volume of water thrown into it. These causes, acting together, cause the flood-water to rise to a height sufficient to overtop and breach the embankment, and then the flood enters the low lands with a rush, devastating the land and flooding the homesteads. The condition of these lands has been receiving the attention of the Bengal Government for many years past, and many proposals have been made for improving their condition, the principal one being to remove portions of the embankment, and admit the river-water freely to the interior lands, which, it was thought, would thus become gradually raised by deposits of silt to a height sufficient to admit of their being successfully drained and cultivated. I cannot here enter into the reasons for and against this proposal. The whole question was exhaustively considered in the year 1900, and the proposal was negatived, mainly on account of the great length of time that would be required to carry it out, and the smallness of the advantages that would result when the operations were completed. The conclusion arrived at was that the most advantageous course would be to drain the circuit. Estimates were accordingly called for and prepared, and I will now explain the principles followed in designing the necessary works.

The works consist mainly of a drainage channel $7\frac{1}{2}$ miles long, aligned as shown in the plan, and discharging into the tidal waters of the Bagda river.

At the outfall a sluice is necessary to exclude the tidal waters when the level of the tideway rises above the level of the water in the channel. Besides these, some minor works are necessary, in the shape of small drainage channels to lead the water into the main channel; and an existing khal has to be cleared of silt. What we are here concerned with is the waterway required for the sluice and the dimensions of the main channel.

The first essential point in this as in all other cases is the determination of the levels, *i.e.*, the ground levels of the lands to be drained, the existing flood level on those lands, and the flood level which we wish to maintain when the works are constructed. Equally important are the levels at the outfall, which were determined by a series of tidal observations, from which the average levels of high and low water were determined. I will now quote from my note, written when the project came before me for revision.

"The ruling conditions are (1) the rate of flow-off to be provided for, (2) the level of low-water in the river, (3) the nature of the tide-curves, and (4) the maximum flood level to be allowed in the circuit. It has been decided to allow a flow-off of $\frac{3}{4}$ inch depth of rainfall over the area in 24 hours, with which I agree. As regards (2) and (3) the data given appear to be correct. As regards (4) some care is required. Mr. Horn in his preliminary rough estimate took 112·00 as the flood level in the circuit. In the present estimate, prepared after the survey had been completed, the flood level in the circuit is taken as 110·30,* *i.e.*, the sluice is considered to cease acting when the tide rises above 110·30. In the detailed plans, the level 110·30 is shown as obtaining at mile $4\frac{1}{2}$, *i.e.*, $2\frac{3}{4}$ miles from the sluice. I cannot quite make out what surface fall is reckoned on from 0 to $4\frac{1}{2}$ miles. From $4\frac{1}{2}$ miles to the sluice the surface-fall is taken as 6 inches per mile, and the *bed*-level of the channel is graded at a uniform slope of that amount from the sluice up to the head. This would imply a surface-level, at the head of the cut, of $(110{\cdot}30 + 4\frac{1}{2} \times {\cdot}5 =)$ 112·55, which I do not think is intended. It is, I think, meant to be implied that the surface fall inside the circuit will be "eased" by a good deal of the discharge flowing over the low ground itself, and not entirely along the channel. In this principle I quite concur. I propose, therefore, to design the channel from 0 to $4\frac{1}{2}$ miles on the basis of a surface slope of 6 inches per mile, but to assume that the actual difference of level between 0 and $4\frac{1}{2}$ miles will be that due to a surface slope of 4 inches per mile. The fall from $4\frac{1}{2}$ miles to the sluice must be taken as equivalent to the actual slope of 6 inches per mile. This ruling gradient (6 inches per mile) is, I think, quite suitable, as anything less would involve a very wide channel, while a greater slope with narrower channel would unduly reduce the head over the sluice, or unduly pond up the water in the remote parts of the circuit. The actual surface levels may, then, be taken as follows:—

At head of drainage cut, *i.e.*, at mile 0	111·80
At $4\frac{1}{2}$ miles	110·30
At sluice	108·68

and the widths of the channel may be calculated on a uniform slope of 6 inches per mile."

We now come to the design of the sluice. First it has to be noted that the soffits of the arches of the vents are designed at the level 107·30, while the level of low water is 107·40, so that the discharge is that of a completely submerged sluice. Next, we must notice that the surface-level in the channel, immediately inside the sluice, is given as 108·68. This, however, refers to the level at low water; and we have to remember that, as the level of the tide-way rises, the surface-level in the channel will also rise, though at a rather slower rate. The flood level in the circuit at $2\frac{3}{4}$ miles from the sluice is 110·30, and it may be taken that the surface of the channel close to the sluice will rise until it reaches the height 110·30, at which point the tide-way will rise above the channel-level, and discharge will cease. It will, I think, give quite a sufficient degree of accuracy to assume† that this reduction of head will occur at a uniform rate; so that we are confronted with the case I have dealt with before, that of a completely-submerged sluice, where the maximum head which occurs at low water is gradually reduced, at a uniform rate, by the rising tide, until it is completely extinguished. Now, looking at the levels given above, we see that the channel is graded so as to give a surface-level of 108·68 at low water, while the low water level of the tide-way is 107·40, giving a working maximum head of 1·28 foot. Thus the

* These levels are referred to a datum 100 feet below mean sea level.

† The accuracy of this assumption should be tested by the method explained in Section IV(b) of these Lectures.

time occupied in extinguishing the head will be the time taken by the tide in rising from the level 107·40 to the level 110·30, which we can obtain from the tide-curve. If you refer to equation (30) and Table IX of these Lectures, you will see that the mean discharge, during the time the head is being extinguished, is $\frac{2}{3}$rds of the maximum head, occurring at low water.

Quoting again from my note; "now, coming to the tide-curves, the shape as drawn seems a little unusual, but if there is an error in this respect, it is on the safe side. The averge time during which the sluice will be closed, owing to the tide standing above the level 110·30, is shown by these curves to be $6\frac{3}{4}$ hours out of the 24, but I think it would be safer to take it that the sluice will work only 16 hours out of the 24, that is, for two-thirds of the time, especially as floods might occur at an unfavourable state of the tide. In the 24-Parganas we usually get sluices to work only 13 or 14 hours, and in the Magra Hát drainage scheme the time allowed was only 12 hours. Thus 16 hours is, I think, as much as should be relied on. The mean discharge for 16 hours will then be $\frac{2}{3}q_1$, where q_1 is the discharge at low water.

"The drainage area is 27 square miles, and a flow off of $\frac{3}{4}$ inch per 24 hours over this area is equivalent to a continuous discharge of ($\frac{3}{4} \times 27 \times 27 =$) 547 c.f.s. lasting for 24 hours, which is equivalent to a mean discharge of ($\frac{3}{2} \times 547 =$) 820 c.f.s. lasting for 16 hours. The required low-water discharge is now given by the equation $\frac{2}{3}q_1 = 820$; that is, $q_1 = 1{,}230$ c.f.s."

The dimensions of the channel can now be fixed. The *data* given above are as follows; surface slope 6 inches per mile; depth 6·68 feet (since the bed-level is designed at 102·00 and the surface, as we have seen, is at the level 108·68); discharge 1,230 c.f.s. From Mr. Odling's tables it is seen that a bed-width of 88 feet will just suffice; and it will be better to make it 90 feet.

In calculating the water-way required for the sluice, we may take into account the effect of velocity of approach, involving the quantity Ω, the area of cross-section of the approach-channel. With the dimensions just determined, the area will be [(90 + 6·68) 6·68 =] 648 square feet. The expression for the discharge of the sluice, involving velocity of approach, given as equation (62) of these Lectures, may be written in the form

$$a = \frac{q}{c\sqrt{2g}\left(h + \frac{q^2}{2g\Omega^2}\right)^{\frac{1}{2}}}$$

the values of the quantities in this case being as follows :—

$$q = 1{,}230;\ c\sqrt{2g} = 6;\ h = 1{\cdot}28;\ \Omega = 648$$

and the resulting value of a is 177 square feet. The sluice is designed with six vents, each having 31·37 square feet of waterway, which gives a total of 188·22 square feet, which is slightly more than sufficient.

As regards the widths of the upper portions of the channel, the full discharge will only be carried by the lower portion, and the volume of drainage in the upper parts will be less. Besides, it is not necessary that the upper portions should be capable of carrying the maximum discharge at low water, on account of the reservoir-like action in the channel itself and in the low lands of the circuit, which will absorb a certain proportion of the flow-off when the levels rise at the sluice. The channel, as revised by me, is designed with a bed-width of 20 feet at the head (mile 0), gradually increasing to 90 feet at mile $4\frac{1}{2}$, and remaining at 90 feet from $4\frac{1}{2}$ miles to the sluice.

IX(b).—CHALLAN BHIL DRAINAGE SCHEME.

The Challan Bhil is the name given to a very large swamp, or rather collection of swamps, in the districts of Rajshahi and Pabna. The area under water is about 421 square miles in the rains, decreasing, as the water dries up, to about 150 square miles in November, and 50 in April and May. The lowest parts of the *bhils* never dry up througout the year. Much of the land is waste, and the greater part of the remainder is cultivated with a special description of rice, which grows in the water, keeping pace with the increasing depth. Some portions are cultivated with *boro*, a hot-weather crop. The *bhil* is traversed by several rivers, the water in which is, in the flood season, laden with silt and sand, in consequence of which the river beds have silted up to a large extent, and they are no longer able, as they once were, to serve the purpose of draining the *bhils* after the flood season is over. These rivers, after traversing the *bhil*, ultimately discharge into the Burral river, near Noornagar, which is there of sufficient section to serve as an efficient drainage outlet. It is now proposed, in order to drain the *bhils*, to excavate a drainage-cut from the centre of the *bhil* to Noornagar. A regulator has to be provided at its head, to exclude silt and prevent the channel from deteriorating, as the rivers have done. The circumstances are as follows:—The *bhils* which have to be drained constitute, in effect, a large reservoir, which receives the discharge brought down by the rivers. During flood time the levels rise, the rivers overflow their banks; and the water accumulates in this reservoir until the level is raised sufficiently high to enable the silted-up beds of the rivers to carry off a volume equal to that entering. As the volume brought in by the rivers decreases, the levels fall, the depth in the old river beds decreases, and they become less and less efficient as drainage outlets. At the low levels necessary to drain the *bhils* the rivers would be quite unable to carry off the volume entoring, as there would be little, if any, depth of water on their beds. Hence the drainage-cut, which it is proposed to make, must serve as a bye-pass for the incoming volume in addition to its legitimate duty of lowering the level of the *bhils*.

Now it will be found that the discharge necessary to pass the incoming volume is much greater than the discharge required to drain the bhils in a reasonable time, so that the size of the channel must depend very largely on the former volume, and it is very important to determine it correctly. What is required is a knowledge of the discharge brought in by the rivers, on each day during the time that drainage is in progress. Unfortunately in this case the observations were incomplete. The discharge at the beginning of drainage (on the 1st November) was determined approximately, but there was very little information as to the rate at which the discharge decreased; and, as might of course be expected, the discharge of the rivers falls rapidly during the months of November, December and January. I have been obliged, therefore, to proceed largely on assumption, and I have drawn up the statement I annexed showing, according to the best information available, what the incoming discharge may be expected to be, on various dates.

Next, we have to deal with the discharge necessary to drain the *bhils*. That is, in this case, we have to find out by what amount the surface of the *bhils* will be lowered in a given time, by a given discharge. Now this of course depends directly on the *area* of the *bhils* at various levels. An accurate knowledge of those areas would involve running contours, at close intervals, all over the *bhils*, which for many reasons could not be done. We know, however, that the area of the *bhils* varies from 150 square miles on the 1st November to 50 square miles in March, and the levels which have been taken enable us to see at what level these areas hold good. We also know the level of the lowest part of the *bhil*, when the area to be drained is *zero*. The areas at intermediate levels are filled in proportionally, and you will find them in column 5 of statement II. One thing more about these areas. You will see in statement II that the area of 150 square miles occurs when the level of the surface of the drainage-channel is 92·50. As a matter of fact, the *bhils* measure 150 square miles in area when the level at the margin is 94·00. I have, however, allowed

for a fall, or slope, of $1\frac{1}{2}$ feet from the margin of the *bhil* to the channel, to allow the water to drain into the channel, so that when the channel-surface stands at 92·50 it is actually draining an area of 150 square miles. The level of the *bhils*, when they measure 50 square miles, is actually 92·00, but I have allowed a slope of 1 foot in this case, so that this area corresponds with a channell-level of 91·00.

In making the calculations, we have to select the dimensions of our channel, such as we think will be likely to be suitable, and then to calculate the time it will take to drain out the *bhils*. Drainage will begin about the beginning of November, when the levels fall low enough, and must be completed in time to allow the land to be cultivated and the crops sown. It is believed that if the swamps are dry by the end of February, these requirements will be met. It would be better to dry them rather earlier, but this would involve greater expense; and it has to be remembered that the higher portions of the swamps will be dry considerably earlier than the very lowest portion, which is really only a very small proportion of the whole area. We have to note further that, although the beds of the old rivers are too silted to be of any use as drainage channels at low levels, they will assist the drainage channel by carrying off a certain amount of discharge at the beginning of drainage, when the levels are high. We have, again, no *data* to show what the extent of this assistance will be, but I have assumed certain figures, which seem probable, and which you will find in column 4 of statement II. Before beginning the actual calculation, it only remains to draw up a table of discharge of the drainage channel, *i.e.*, a table showing what the discharge will be at various depths. In column 3 of statement II you will find the discharges corresponding with the depths shown in column 2, which occur when the surface stands at the levels shown in coloumn 1. The dimensions selected are as follows; longitudinal slope ·44 feet per mile, *i.e.*, ·083 per 1000; bed-width 100 feet; and side slopes, as usual, 1 to 1.

We can now proceed with the calculations. As the conditions (discharges of rivers and channel, and area of *bhils*) are continually changing, it is necessary to divide the period of drainage into small intervals, during each of which the conditions may be assumed constant. Periods of five days each will give quite sufficiently accurate results. Looking at statement III, in the top line, write down in column 1 the level of the surface of the channel when drainage begins, viz., 92·50. From statement II we see that, at this level, the discharge of the drainage channel is 4,273 c.f.s. and the discharge carried off by the rivers is 1,000 c.f.s. Entering these figures in columns 2 and 3, the total *out-going* discharge (entered in column 4) is 5,273 c.f.s. Now the *in-coming* discharge brought in by the rivers is 14,000 c.f.s. on the 1st November, and it is obvious that no drainage can begin until this in-coming discharge falls below 5,273 c.f.s., say, to 5,000 c.f.s. Statement I shows that this will occur about the 27th December; but this date is a little uncertain, owing to the imperfect nature of the *data*, and in any case will vary slightly every year. We calculate, then, our time of drainage from the date on which the in-coming discharge falls to 5,000 c.f.s. Entering this figure in column 5, and substracting it from the figure in column 4, we have a balance (column 6) of 273 c.f s., which is the excess of the out-going over the in-coming discharge, and consequently represents the discharge available for draining out the water accumulated in the *bhils*. Now a discharge of 27 c.f.s. will lower 1 square mile by 1 inch in one day, so that the rate of lowering per day of M square miles by q c.f.s. is given by the formula r inches $= \frac{q}{27M}$; that is, divide the discharge in column 6 by 27 times the area in column 7. The result is shown in column 8 in inches, and the lowering in five days is shown in feet in column 9. The figure in this line is ·028, so that at the end of five days the surface of the channel (assuming the channel to fall concurrently with the *bhil*) will be (92·50—·028=) 92·472, as shown in column 10. For the second period of five days, enter the surface level 92·47 in column 1, in the second line, and from column 3 of statement 2 obtain by proportional parts the discharge of the channel, viz., 4,257, and enter it in column 2. In column 3 is shown the discharge carried off by the rivers, obtained by proportional parts from column 4 of statement II, viz., 986 c.f.s. The sum of these two, viz.,

5,243 c.f.s , is shown in column 4 as the total out-going volume. The in-coming discharge, by proportional parts from statement 1, is shown in column 5, and the balance available for draining the *bhil* is given in column 6 as 743 c.f.s. The area of the *bhils* at the level 92·47 is found from column 5 of statement II to be 14·8 square miles and the depth run off in five days is found as before, and shown in column 9 as ·08 feet. Thus the level of the channel at the end of the second period of five days is (92·47—·08 =) 92·39, as shown in column 10 of statement III. I need not go through the whole of the calculations, which are made in exactly the same way, with the results shown in statement III. It will be seen that to drain the *bhils* to a level of 86·14 will occupy seventeen periods of five days, *i.e.*, 85 days in all. No allowance has been made for evaporation, so that practically the time would be somewhat shorter, unless there was any rainfall. If drainage began on the 27th December it would, on this calculation, be completed by the beginning of March, and the greater part of the *bhils* would be dry much earlier.

STATEMENT I.

Showing discharge brought in by the rivers.

Date.	1st November.	10th November.	20th November.	30th November.	10th December.	20th December.	30th December.	9th January.	19th January.	29th January.	8th February.	18th February.	28th February.	10th March.	20th March.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Discharge ...	14,000	12,000	10,000	8,500	7,000	5,750	4,750	3,750	3,000	2,500	2,100	1,700	1,350	1,000	750

STATEMENT II.

Level of channel at head.	Depth of channel.	Discharge of channel.	Discharge carried off by rivers.	Area of *bhils*.
1	2	3	4	5
R. L.	Feet.	C. f. s.	C. f. s.	Sq. miles.
92·50	13·25	4,273	1,000	150
92·00	12·75	4,001	765	115
91·50	12·25	3,739	560	80
91·00	11·75	3,485	375	50
90·50	11·25	3,237	220	44
90·00	10·75	2,996	95	38
89·50	10·25	2,764	Nil	32
89·00	9·75	2,504	Nil	26
88·00	8·75	2,085	Nil	20
87·00	7·75	1,694	Nil	14
86·00	6·75	1,338	Nil	8
85·00	5·75	1,013	Nil	2

STATEMENT III.

Level of channel at head.	Discharge of channel.	Discharge carried off by rivers.	Total outflow (2+3).	Discharge brought in by rivers.	Discharge available for lowering *bhils* (4—5).	Area of *bhils*.	Rate of lowering per day.	Height lowered in five days.	Level of channel at end of five days (1—9).
1	2	3	4	5	6	7	8	9	10
R. L.	C. f. s.	C. f. s.	C. f. s.	C. f. s.	C. f. s.	Sq. miles.	Inches.	Feet.	R. L.
92·50	4,273	1,000	5,273	5,000	273	150	·067	·028	92·472
92·47	4,257	986	5,243	4,5 0	743	148	·19	·03	92·39
92·30	4,213	948	5,161	4,000	1,161	142	·30	·125	92·265
92·265	4,145	889	5,034	3,562	1,472	134	·41	·17	92·095
92·10	4,055	812	4,867	3,187	1,680	122	·51	·21	91·89
91·89	3,943	720	4,663	2,875	1,788	107	·62	·26	91·63
91·63	3,807	618	4,420	2,625	1,795	89	·75	·31	91·32
91·32	3,648	493	4,141	2,400	1,741	69	·93	·39	90·93
0·93	3,450	353	3,803	2,2 0	1,603	49	1·21	·50	90·43
90·43	3,203	216	3,419	2,025	1,394	43	1·20	·50	89 93
89·93	2,964	82	3,046	1,825	1,221	37	1·22	·61	80 42
89·42	2,722	Nil	2,722	1,035	1, 87	31	1·30	·54	88·88
58·88	2,454	Nil	2,454	1,460	994	26	1·41	·59	88·29
88·29	2,210	Nil	2,210	1,285	925	22	1·56	·65	87·64
87·64	1,944	Nil	1,944	1,110	834	18	1·72	·72	86·92
86·92	1,666	Nil	1,666	954	712	14	1·88	·78	86·14
86·14	1,388	Nil	1,388	829	559	9	2·30	·96	85·18

IX (c).—CHOROIKUL SWAMP.

This swamp is situated near Kumarkhali, in the Kushtia subdivision of the Nadia district, and is probably one of those low-places which escaped the action of land-raising carried on by the rivers, and has now been left at a permanently low level. It is, in any case, a permanent swamp, where crops cannot be grown, and it is desired to drain it out, so as to enable it to be brought under cultivation. It is not required to construct drainage works to carry off the water that inundates the land during the rainy season. That is accepted as a necessary evil, or at least as one whose remedy is beyond the financial resources of the district. All that it is proposed to do is to drain out the swamp after the rains are over, so as to enable the cultivators to sow winter crops on the higher lands, and "boro" rice on the lower lands, later in the year.

The swamp is situated near the Gorai river, and it is proposed to excavate a drainage cut leading into that river. Close to the swamp runs a line of railway, in which there is a bridge with a span of 14·25 feet clear, and the water must be led through this bridge, so that the drainage-cut extends from the bridge to the Gorai river. Between the railway and the river, again, there runs a District Board road, which will have to be provided with a bridge to carry it over the proposed drainage-cut. Now the first question that arises is, whether the drainage-cut, if constructed, will be liable to silt up and whether it is necessary to construct a sluice to exclude the silt, and also to exclude floods from the river. Enquiries made by the Executive Engineer seem to show that the floods of the Gorai river do not overtop its bank and enter the swamp. If, however, a cut is made, there would be a chance of the floods entering through the cut; and hence it will be advisable to construct some sort of masonry work to exclude them. As regards silt, there will be a very good fall in the cut, towards the river, and there will probably be more danger of scouring than of silting; but if any silt *were* deposited, the villagers are prepared to clear it out themselves every year, so that no expense need be anticipated from that cause. The level of the swamp, when drainage will be in progress, will be so much above that of the river, that we can choose any slope of bed that we wish. A slope of 9 inches per mile has been selected, as any higher slope might cause scour of the bed and banks, the soil being friable and soft. As, however, the level of the river will be much below the level we wish to maintain at the tail end of the drainage-cut, it will be necessary to construct a regulator, to hold up the level in the cut, and prevent excessive velocity. Thus the regulator will serve the double purpose of excluding floods from the river in the rainy season, and of maintaining the proper level in the cut during drainage. The next question is, where to locate the regulator. The banks of the river are unstable and cutting, and the soil is friable, so that any work situated on the river bank would be liable to be washed away. The site selected is about halfway between the railway bridge and the river. Between the regulator and the river, it is probable that scour will occur, but it will be in a place where it does not matter.

In making our calculations as to the time of drainage we have to remember first that the discharge has to be passed through the railway bridge, which has a rather small waterway, and that consequently there will be a considerable loss of head (*i.e.*, a drop of surface level) at the bridge. But now it will, I think, be best to quote from the report I made on the subject, modified in one point after subsequent consideration. The final recommendations are as follows.

The Executive Engineer's report may be accepted as regards the following points:—(*a*) It is not required to drain the *bhil* during the rains, but to lower it nearly to the level of the existing floor of the railway bridge, within a reasonable time after the rains are over. (*b*) A sluice is not necessary or desirable, for the reasons given. (*c*) Some measures are necessary to exclude

floods from the *bhil.* (*d*) The size of channel proposed by the District Engineer is quite inadequate.

As regards (*a*), the existing floor of the bridge should be lowered, for the reason that, as the level of the swamp falls to nearly the same level as the bridge-floor, the discharge over the floor will be very small, and the last foot of drainage will take a very long time. The bottom of the swamp is 2 feet below the existing floor, and I consider it essential that the floor should be lowered to this level, *i.e.*, by 2 feet.

As regards (*b*) and (*c*), although a sluice on the river bank is not desirable, it would be advisable to provide a regulator in the channel about midway between the railway and the river, to regulate the level in the channel, and prevent any scour near the head of the channel. The lower part of the channel may be left to scour if it likes. The regulator can also be boarded up during the flood season, so as to prevent the floods from the river from entering the *bhil.* Further, the bridge on the District Board road must be given plenty of waterway so as to cause no heading up of level in the channel. Next, as to the size of the channel, I think the channel should have a bed-width of 30 feet, with side-slopes of 1 to 1, and it may be given a longitudinal bed-slope of 9 inches per mile. The channel will be "fed" by the discharge through the railway bridge, and the two discharges (of bridge and channel) must be considered together. The discharges of the channel at various depths are shown in statement I attached. Now, supposing the floor of the bridge is lowered 2 feet and the bed of the channel is 1 foot below that again, the conditions will be as shown in the diagram attached.*

Assuming, as a trial value, that the head over the bridge (*i.e.*, the difference of level between the surface of the swamp and channel) will be 1 foot, then the depth in the channel will be 7 feet, and the discharge through the bridge will be

$$q = c\sqrt{2g}\ b\ d\ h^{\frac{1}{2}} + \frac{2}{3} c\sqrt{2g}\ b\ h^{\frac{3}{2}}$$

$$= \frac{2}{3} c\sqrt{2g}\ b\ h^{\frac{1}{2}} \left(\frac{3}{2} d + h\right)$$

The coefficient applicable to the sluice portion of the discharge is about $c\sqrt{2g} = 6$. In the weir portion there is no contraction, and the coefficient should be at least as high as for the sluice portion. Using these values, we have

$$q = 4\ b\ h^{\frac{1}{2}} \left(\frac{3}{2} d + h\right)$$

That is, since $b = 14{\cdot}25$,

$$q = 57\ h^{\frac{1}{2}} \left(\frac{3}{2} d + h\right)$$

Now in this case our trial value of h is 1, so that $q = 570$ c.f.s. Looking at statement I we see that this is almost equal to the channel discharge at a depth of 7 feet, viz., 562. This trial value therefore holds good, and the "head" over the bridge will be just 1 foot.

Next, when the level in the swamp has fallen to such an extent that the depth in the channel is 6 feet, we shall have the channel discharge 428 c.f.s. and the bridge discharge $57\ h^{\frac{1}{2}} (7{\cdot}5 + h)$. Thus we have to determine h,

$$428 = 57\ h^{\frac{1}{2}} (7{\cdot}5 + h)\ ;\ i.e.,\ h^{\frac{1}{2}} = \frac{7{\cdot}5120}{7{\cdot}5 + h}$$

From this we find by trial that $h = {\cdot}82$, very nearly. Then the level of the swamp will be 5·82 feet above the (lowered) floor.

Next, when the channel depth is 5 feet, we have in the same way as above :—

$$313 = 57\ h^{\frac{1}{2}} (6 + h)\ ;\ h^{\frac{1}{2}} = \frac{5{\cdot}4913}{6 + h}$$

By trial h is found to be $= {\cdot}68$ feet, and the level of the swamp will be 4·68 feet above the (lowered) floor.

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* *See* diagram facing page 82.

When the depth of the channel is 4 feet we shall have $d = 3$ and $q = 214$, and to determine h

$$h^{\frac{1}{2}} = \frac{214}{57(4\cdot5+h)} = \frac{3\cdot7544}{4\cdot5+h}$$

By trial h is found to be $= \cdot55$, and the level of the swamp will be 3·55 feet above the (lowered) floor.

When the depth of the channel is 3 feet, the value of d will be 2 and of q, 130, and we have

$$h^{\frac{1}{2}} = \frac{2\cdot2807}{3+h}$$

By trial h is found to be $= \cdot44$, and the level of the swamp will be 2·44 feet above the (lowered) floor.

When the depth of the channel is 2 feet, d will be 1, and $q = 64$, and

$$h^{\frac{1}{2}} = \frac{1\cdot228}{1\cdot5+h}$$

By trial h is found to be$=\cdot36$, and the level of the swamp will be 1·36 feet above the (lowered) floor.

This is quite a sufficiently low level to drain the swamp to, and we may now calculate how long the drainage will take. The calculation depends on the area of the swamp, which will of course become less and less as the level falls. No data as to the areas at different levels (other than full level) are available, so they have to be assumed. The area at full level is 7·5 square miles, and it will probably be on the safe side to assume that when the depth of the swamp has been decreased from 7 feet to 1·36, the area will not exceed 2·5 square miles. The areas at intermediate depths are filled in arbitrarily. We can now fill up columns 1 to 5 of statement II, showing the discharges and areas at the different depths.

The time of drainage, shown in the last column of the statement, is calculated as follows. It is known that a run-off of r inches in depth from M square miles is equivalent to a continuous discharge of $\frac{27Mr}{x}$ c.f.s. lasting for x days. Thus we have

$$x = \frac{27Mr}{q}$$

Now, while the swamp is being lowered from 7 feet to 5·82 feet, we have the mean discharge $q=\frac{1}{2}$ $(562+428)=495$ c.f.s. The mean area M is$=7$ square miles; and the run-off 1·18 feet=14·16 inches.

Hence—

$$x=\frac{27\times7\times14\cdot16}{495}=5\cdot406 \text{ days.}$$

For the second period, from 5·82 to 4·68, we have $q=370$; $M=6$; $r=1\cdot14$ feet$=13\cdot68$ inches..

Hence—

$$x = \frac{27\times6\times13\cdot68}{370}=5\cdot9896 \text{ days.}$$

For the third period, from 4·68 to 3·55, we have $q=263$; $M=5$; $r=13\cdot56$ inches, and

$$x = \frac{27 \times 5 \times 13\cdot56}{263} = 6\cdot9603 \text{ days.}$$

For the fourth period, from 3·55 to 2·44, we have $q = 172$; $\text{M} = 4$ $r = 13\cdot32$ inches, and

$$x = \frac{27 \times 4 \times 13\cdot32}{172} = 8\cdot3635 \text{ days.}$$

For the fifth period, from 2·44 to 1·36 we have $q = 97$; $M = 3$; $r = 12·96$ inches, and

$$x = \frac{27 \times 3 \times 12·96}{97} = 10·822 \text{ days.}$$

The above figures are entered (approximately) in the last column of the table.

Thus the swamp will drain nearly down to the existing floor level of the bridge in 27 days, and down to 8 inches below the existing floor level in 38 days. This will, if anything, more than meet the requirements of the cultivators. The maximum velocity through the bridge will be about 6 feet per second, and there will be a drop of 1 foot to the bed of the channel. To avoid scour, a masonry apron below the bridge will be necessary. Summing up, the alterations necessary are as follows:—

(1) The floor of the railway bridge must be lowered by 2 feet, and an apron provided.
(2) The dimensions of the channel must be; 30′ bed-width; side-slopes 1 to 1; longitudinal slope 9 inches per mile.
(3) The bed of the channel, at the head, to be 3 feet below the existing bridge-floor.
(4) The district road bridge must have ample waterway, so as to cause no obstruction or heading-up.
(5) A regulator should be provided about midway between the railway and the river.

The estimate should be remodelled accordingly.

Statement I.

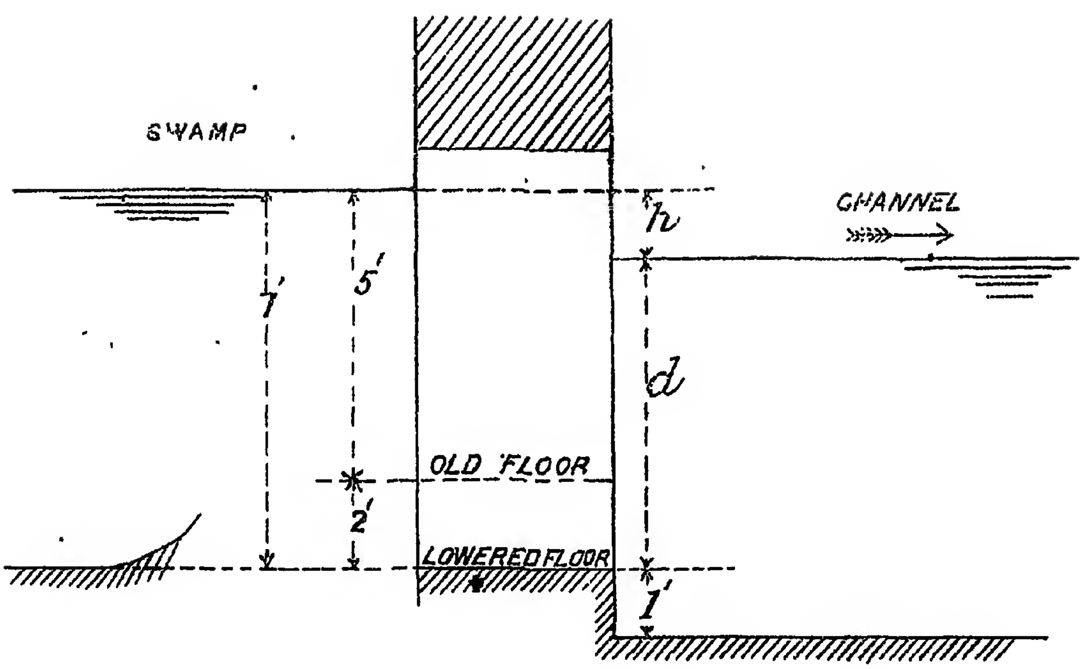
Showing discharges of channel at various depths.

Depth = ...	2′	3′	4′	5′	6′	7′
Discharge = ...	64	130	214	313	428	562

Statement II.

Showing areas and discharges at different depths, with time of drainage.

Depth of swamp.	Area of swamp.	Depth of channel.	Head over-bridge.	Discharge of channel.	Time of drainage.
1	2	3	4	5	6
Feet.	Sq. miles.	Feet.	Feet.	C. f. s.	Days.
7·00	7·5	7	1·00	562	5½
5·82	6·5	6	0·82	428	6
4·68	5·5	5	0·68	313	7
3·55	4·5	4	0·55	214	8½
2·44	3·5	3	0·44	130	11
1·36	2·5	2	0·36	64	
				Total ...	38



SWAMP
CHANNEL
h
5'
7'
d
OLD FLOOR
2'
LOWERED FLOOR
1'

IX(d).—MAGRA HÂT DRAINAGE SCHEME.

The calculations for this scheme are based mainly on the principles pointed out in Sections IV(*b*) and VII of these Lectures, so that it will constitute a good practical example. It will, I think, only be necessary to quote the report on the revised rough estimates for the Magra Hât portion of the scheme, which runs as follows:—

The removal of the outfall sluice from Usti to Diamond Harbour increases the length of the outfall channel by about 9 miles, without any compensating advantage in level, as the tide levels at both places are the same. This causes a flattening of longitudinal slope, and necessitates wider channels. Again, since the levels for the scheme were taken, the Diamond Harbour Creek has silted up rapidly. From Usti to Bendal the average rise of level of the bed has amounted to 6 feet in the $2\frac{1}{2}$ years that have elapsed. Both these causes point to a considerable increase in cost, and necessitate a revision of the calculations.

2. In the former calculations the depth of water passing over the weir-wall of the sluice was taken as 4 feet, in calculating the breadth of weir required. In order to gain as much slope as possible, it may now be taken as 3 feet only, and the wing-walls splayed out to give the requisite breadth of weir. For the same reason the crest of the weir is lowered from + 2·00 to + 0·00 M. S. L. This fixes + 3·00 as the level of the surface (during free discharge, at low water) at the tail of the channel. The level at the head of the outfall depends on the flood-level over the country. First consider the high flood-level. In the previous report, the surface level of the channel at Nainan (the head of the main outfall) was taken as 6·4, which, allowing a slope of 0·264 feet per mile to lead the water from the north part of the Kaorapokhar basins to Nainan, gave a flood-level of +8·517 three miles north of Andharmanik. With these levels, a rate of a flow of $\frac{1}{4}$ inch in 24 hours was allowed for the channels leading to Nainan, as most of the flow would occur over the ground itself. These subsidiary channels may remain as designed on that basis, but in designing the main outfalls it is not now practicable to work to such low levels. In the flood of 1901 the levels at the sluices in the embankments varied from +10·00 to +12·00. The height 10·50 may be taken as the full flood-level and a rate of run-off of $\frac{3}{4}''$* per 24 hours may be allowed. With the levels at that height, the whole country would be under water, and no slope need be allowed for from the remoter parts of the swamps. The ground level in the Kaorapokhar basins is mostly between +6·00 and +7·00. It is a little higher towards Surjipur. In the Sangrampur basin, on both sides of the Railway, it varies from about +7·00 to +8·00, with only one or two lower places, of small area. The lowest level is in the great Joynagar swamps, which run from about +5·00 to +6·00. Thus, for a medium flood-level, a height of + 9·00 will not give an undue depth over the ground. The lowest parts will be drained after the top of the flood has run off. Two sets of calculations are made, both allowing a depth of 3′ over the weir, with cut-off at 7′, viz., (1) for a flood-level of 10·50 at Nainan and in the Joynagar swamp, with a run-off of $\frac{3}{4}''$* per day, and (2) for a flood-level of 9·00, with a run-off of $\frac{1}{2}''$.†

3. In the previous scheme the drainage from the Joynagar swamps was taken round *viâ* Nainan, but, as the waterway in the Railway bridge at Magra Hât is small, it will be more economical to conduct this drainage through the Sangrampur cut, as there is plenty of waterway in the bridge over the Nazra khal.

4. The efficiency of the sluice depends largely on the level of "cut-off," *i.e.*, on the level at which the surfaces inside and outside the sluice coincide and discharge ceases. Unfortunately this has not been previously recorded in our observations at drainage sluices, and the information will take time to collect. In the present case it will be assumed that discharge ceases (on a rising

* The actual rate of run-off will be ·609 inches.
† Ditto ditto ·406 do.

tide) when the levels stand at +7·00. On a falling tide the level will be higher, as the water inside has been collecting and ponding up at the tail of the channel during high tide, but it will be on the safe side to take it at +7·00 in both cases. The conditions are now as follows. As the tide rises above the weir crest, the discharge becomes "drowned," but the level inside the sluice rises at the same time, until the levels coincide and discharge ceases. That is, the tide rises from 0·00 to +7·00 and the channel surface rises from +3·00 to +7·00 at the same time.

5. The mean discharge during the period of "drowning" is a function of the discharges of both sluice and channel, and its determination is effected partly by analytical and partly by graphic methods, as shown below. The fomulæ for q_o, the initial discharge, and b_w, the width of the weir, are derived from numbers 33, 34, 35 and 36 at page 17 of my paper on "Flood Drainage," but with a slight modification, making them of more general application (see equations 78, 79, 80, 81 of Section VII of these Lectures).

The notation used is as follows:—

M=area of drained tract, in square miles.
r=rate of flow-off per 24 hours, in inches.
t_o=time of "free" discharge, in hours.
t_1=time of partially drowned discharge, in hours.
q_o=rate of "free" discharge.
q_m=mean discharge during time t_1.
q_M=maximum discharge during time t_1.
q_{mm}=mean discharge during whole tide.
b_w=width of weir.
x_o=depth of flow over weir-crest at low tide.

And the formulæ are as follows:—

$$(a)\ q_o=\frac{350Mr}{t_o+\frac{q_m}{q_o}t_1}$$

(b) $\frac{q_m}{q_o}$ is determined by the methods mentioned above.

$$(c)\ b_w=\frac{3}{10}\frac{q_o}{x_o^{\frac{3}{2}}}$$

$$(d)\ \frac{q_{mm}}{q_o}=\frac{t_o+\frac{q_m}{q_o}t_1}{13}$$

6. As a preliminary trial, the outfall channels and weir were designed to suit a discharge of 5,115 c. f. s. at low tide, with a weir breadth of about 295 feet. Two sets of calculations were made, shewn in Appendices II, III, IV, and V. At that time the value of $\frac{q_m}{q_o}$ had not been properly determined, and the two sets of figures were assumed to represent rates of run-off of $\frac{1}{2}''$ and $\frac{3}{4}''$ respectively. These rates have now to be modified; but it is believed the scheme will be of sufficient capacity as designed, and the calculations will indicate the actual rates of flow-off which the different parts of the scheme are capable of carrying.

7. The channels were designed as follows. The first thing was to determine the drainage area served by each channel. This is shewn in detail in Appendix I annexed. Appendix II shews the discharges in each channel, calculated at different rates of run-off. The values of q_o and q_{mm} are shewn for the outfall channels, reduced, for the upper reaches, proportionately to the areas served. The dimensions of the channels are selected so as to give discharges as nearly up to the maximum requirements as is practicable and suitable. The discharges worked to are shown in antique type in this statement.

8. The detailed calculations for the sizes and levels of the outfall channels are shown in Appendices III and IV. The bed-levels selected are the lowest, and the width the greatest, resulting from these two sets of calculations. The details of the remaining channels are calculated in Appendix V, on the basis

adopted in the previous estimate. As regards the outfall channels, the channels are proportioned so as to give flatter slopes, with wider beds, in the lower part of the creek, where a good channel already exists, and comparatively little excavation is required; reserving the steeper slopes and smaller cross-sections for the higher ground, as far as possible. In the Sangrampur outfall, the levels and length of channel necessitate a low slope all the way. The diversion cut at Diamond Harbour is through such high ground that it would be extremely expensive if a high slope were not allowed, and the short length of the cut allows of this being done.

In the higher reaches, the effect of tidal action is less, there is less difference between the maximum and mean discharges, and the required discharges will be nearer those calculated on the basis of a continuous run-off.

The cost of the channels is calculated in Appendices VI and VII, which explain themselves. The "mean level of ground or khal" for the Diamond Harbour Creek was calculated from the cross-sections just completed, by plotting on them a channel with bed-width varying from 300′ to 160′, and taking out the actual quantities. For the smaller bed-widths as now designed there will be less excavation at the sides, and the true bed-level of the creek will work out lower than shown, resulting in some saving.

9. It may now be enquired what the actual rates of run-off will be, with the dimensions of weir and channels as designed. The actual width of weir has been made 300 feet, but in the calculations the figure 290·64 will be used, as it corresponds with the discharge 5,138 c. f. s. which is the figure given by these calculations. First of all, a curve has to be plotted showing the discharges of the channel at different depths. This is shown in Appendix VIII. From this curve the discharge at any intermediate depth can be found by scale, if required. The discharges are calculated from the formula—

$$Q=100CA\sqrt{RS}$$

and the values of C are obtained from Jackson's Table II, for the value $N=\cdot0250$, using proportional parts. The remaining quantities are calculated. The calculations are as follows:—

At tail, channel level varies from + 3·00 to + 7·00
,, ,, depth ,, ,, 10′ to 14′
Length of channel = 16 miles
Total fall, at low water=10·50—3·00=7·5 feet.
Mean slope, at low water$=\frac{7\cdot5}{16}=0\cdot47$ feet per mile.
It is assumed that discharge ceases at level + 7·00.
At low water, the level 7·00 is at $(\frac{4}{7\cdot50}\times16=)$ 8·53 miles from sluice.
Then, for various depths, the quantities are as follows:—

		10′	11′	12′	13′	14′
Depth=		10′	11′	12′	13′	14′
x=		3′	4′	5′	6′	7′
Mean slope=	feet per mile.	0·47	0·3525	0·235	0·1175	0·000
Slope in diversion channel	feet per mile.	1·056	$\frac{\cdot3525}{\cdot47}\times1\cdot056$	$\frac{\cdot235}{\cdot47}\times1\cdot056$	$\frac{\cdot1175}{\cdot47}\times1\cdot056$	0·000
	per 1,000	·20	·15	·10	·05	·00
Bed-width=		130	130	130	130	130
Area= A=		1,400	1,551	1,704	1,859	2,016
Wetted perimeter=P=		158·28	161 11	163·94	166·76	169·59
$R=\frac{A}{P}$=		8·8451	9 6270	10·394	11·148	11·888
S=		·20	·15	·10	·05	·00
N=·025; C= ... ... (from Jackson's Table II).		·87262	·89322	·92247	·98266	—
$Q=100\,CA\sqrt{RS}$=		5,138	5,264	5,068	4,913	0

From these results the curve shown in Appendix VIII is plotted.

10. The weir discharges are most conveniently obtained from Table V of my Lectures, because that formula gives the discharge in terms of x, and not of h. The Table is reproduced as Appendix IX. The height x is measured from the weir crest, and its value at different channel depths has been shown in the foregoing statement. Using the corresponding values of x and q therein shown, we have next to calculate the corresponding values of F from the formula—

$$F=\frac{q}{c\sqrt{2g}\,bx^{\frac{3}{2}}},\ \text{where } c\sqrt{2g}=5;\ b=296{\cdot}64,$$

and then from the Table in Appendix IX to find what values of $\frac{y}{x}$ correspond with these values of F, and thence compute the corresponding values of y. Since we are dealing with a weir, the quantity $\frac{D}{x}$ is equal to 1, and the last column of the Table is applicable. The results are shown below:—

$x=$	3′	4′	5′	6′	7′
$q=$	5138	5264	5068	4313	0
$F=q\div1475x^{\frac{3}{2}}=$...	·66607	·44611	·30733	·19896	0
$\frac{y}{x}=$	0	·7620	·8986	·9570	1·0
$y=$	0	3·05	4·49	5·74	7·00

11. We have now calculated corresponding values of x, q, and y, and since y, in the case under notice, is proportional to the time that has elapsed since "drowning" began, it follows that if a curve be plotted with values of y as abscissæ and values of q as ordinates, the true mean value of q during the time of "drowning" will be given by the area of this curve, divided by the maximum abscissa (*i.e.*, 7 00); and the value of $\frac{q_m}{q_o}$ is then obtained directly.

In the diagram (Appendix X), the value of $\frac{q_m}{q_o}$ is the ratio of the curve *OABCDE* to the rectangle *OAFE*.

(It may be noted that if the tide did not rise at a uniform rate, the corresponding values of y and t would have to be obtained from the tide-curve, and the values of t plotted as abscissæ to the q-ordinates, instead of the values of y.)

In the present case the area of the curve is as follows:—

	3 05 × 5,245	= 15,997
	1·44 × 5,210	= 7,502
	1·25 × 4,730	= 5,912
	0·26 × 4,160	= 1,082
	1·00 × 2,850	= 2,850
Total ...	7·00 × 4,753	= 33,343

Dividing 33,343 by 7, we get the value of the mean ordinate, q_m, as 4,763; and since the initial discharge q_o is 5,138, the value of the required ratio $\frac{q_m}{q_o}$ is $\left(\frac{4,763}{5,138}=\right)$ ·927.

The maximum discharge is about 5,264, occurring when $\frac{3}{7}$ths of t_1 has elapsed.

12. Now returning to the formula (a) of paragraph 5 above the actual rate of run-off is found from the expression—

$$r = \frac{q_o\left(t_o + \frac{q_m}{q_o} t_1\right)}{350M}$$

It is found by scale from the tide-curve given as Appendix II of my former report, dated the 5th September 1902, that $t_o = 4{\cdot}1$ hours and $t_1 = 5{\cdot}2$ hours. This gives, for the value of r—

$$r = \frac{5{,}138\ (4{\cdot}1 + {\cdot}927 \times 5{\cdot}2)}{350 \times 215} = \frac{5{,}138 \times 8{\cdot}92}{350 \times 215}$$

$$r = {\cdot}609$$

That is, the run-off is at the rate of nearly $\frac{5}{8}$ths of an inch in 24 hours instead of $\frac{3}{4}''$ as at first assumed. This rate is probably sufficient.

The lower rate of run-off used in the preliminary calculations will now be $\left(\frac{{\cdot}609}{{\cdot}750} \times {\cdot}5 = \right){\cdot}406$, instead of $\frac{1}{2}''$ as at first assumed.

The dimensions of the channels may remain as designed. They are shown, with the estimated cost, in Appendices VI and VII.

APPENDIX I.

Statement showing drainage areas served by each channel.

Drainage basin.	Area.	Channels in Roman figures, with areas served in antiquo figures.						
Kaorapokhar khal I ...	24	6—6_1 24	6_1—7 42					
Kaorapokhar khal II ...	18			7—4 66				
Hotar khal ...	14	8—7 14						
Kaorapokhar khal III ...	10				4—5 113			
Surjipur khal ...	25	9—10 25	10—4 27					
Kaorapokhar khal IV ..	2							
Main outfall channel ...	20							
Joynagar khal ..	14	14—12 14	12—13 24				5—20 191	
Katta khal ...	10	11—12 10						20—0_1—0 215
Dhanpota khal ...	1			13—17 27	17—18 36			
Sangrampur khal ...	2					18—16—5 50		
Dhanpota khal ...	5	15—17 5						
Sangrampur khal ...	4							
Sangrampur khal ...	14							
Srichandra khal ...	28	1—5 28						
Bendal, etc. ...	14							
Creek	10							

APPENDIX II.

Statement showing discharges of each channel at different rates of flow-off.

NOTE.—The selected discharges are shown in heavier type.

CHANNEL.		Area served.	DISCHARGE. Allowing for full tidal action. At ½″ run-off; (1) 3′ over weir; cut off at 7′.		At ¾″ run-off; (2) 3′ over weir; cut off at 7′.		At continuous run-off. ¾″.	½″.	¼″.
1		2	3	4	5	6	7	8	9
			q_o	q_{mm}	q_o	q_{mm}			
Diversion	...	215	**3,410**	2,894	5,115	4,341	4,330	2,890	1,445
Creek	0—0₁	215	**3,410**	...	5,115	...	4,336	2,890	1,445
Creek, to Bendal	0₁—20	215	**3,410**	...	5,115	...	4,330	2,890	1,445
Creek, Bendal to Usti	20—5	191	**3,020**	...	4,544	...	3,882	2,568	1,284
Outfall, Usti to Nainan	5—4	113	**1,792**	...	2,688	...	2,270	1,518	750
Creek, Nazra Khal	5—10	50	**793**	...	1,190	...	1,008	672	336
Sangrampur outfall	10—18	50	**793**	...	1,190	...	**1,008**	672	336
Ditto	18—17	36	**571**	...	857	...	**720**	484	242
Kaorapokhar khal	4—7	66	...	...	...	...	1,331	862	**444**
Kotar khal	8—7	14	...	...	...	...	282	188	**94**
Kaorapokhar khal	7—0₁	42	...	...	...	...	847	564	**282**
Ditto	6₁—0	24	...	...	...	...	484	322	**161**
Ditto, connecting channel	4—10	27	...	...	...	...	545	364	**182**
Surjipur khal	10—9	25	...	...	...	...	504	336	**168**
Jaynagar and katta khals, and outfall	17—13	27	**428**	...	...	...	**545**	364	182
	13—12	24	**381**	...	...	...	**484**	322	161
	12—14	14	...	...	...	...	282	188	**94**
	12—11	10	...	...	...	...	202	134	**67**
Dhanpota khal	17—18	5	...	...	...	...	101	68	**34**
Srichandra khal	6—1	28	...	...	...	...	**565**	376	**188**
Connecting channel	18—10	...	...	...	...	...	...	...	...

(1) NOTE.—The actual rate of run-off will be ·609 inches.
(2) Ditto ditto ·400 do.

APPENDIX III.

Statement showing levels and sizes of channels to suit ½″(1) run-off, with 3′ depth over weir, and cut-off at 7′, and 9·00 flood level in swamps.

CHANNEL LETTERED.	Required discharge.	Level of surface	Longitudinal slope of surface. Per 1,000.	Feet per mile.	Depth.	Bed width.	Calculated discharge.	Length.	Total surface fall.	Resulting surface level.	Resulting bed level.	Selected bed level.	REMARKS.
1	2	3	4	5	6	7	8	9	10	11	12	13	14
					Ft.	Ft.							
Diversion	3,410	3·00	0·15	0·792	10	100	3,462	⅝	0·495	3·000 3·495	−7·00 −6·50	−7·00 −7·00	
0 0₁	3,410	...	0·05	0·264	10	170	3,602	1½	0·306	3·495 3·801	−6·50 −6·11	−7·00 −7·00	
0₁ 20 (Bendal)	3,410	...	0·05	0·264	10	170	3,602	3½	0·924	3·801 4·815	−6·11 −5·18	−7·00 −9·00	
20 (Bendal) ... ... 5 (Usti)	3,029	...	0·08	0·4,224	10	120	3,084	3¾	1·554	4·815 6·399	−5·19 −3·00	−9·00 −5·00	
5 (Usti) ... ... 4 (Nainan)	1,792	0·145	0·08	0·4,224	9	80	1,717	9½	2,740 0,145	6·399 0·145	−2·60 +0·15	−4·00 −1·00	
5 (Usti) ... ... 10 (Sangrampur)	793	...	0·05	0·264	9	50	852	2½	0·660	6·399 7·059	−2·60 −1·94	−3·00 −2·34	
10 18	793	...	0·05	0·264	9	50	883	4	1·056	7·059 8·115	−1·94 −0·88	−2·34 −1·28	
18 17	571	...	0·05	0·264	8	40	577	2	0·528	8·115 8·643	+0·12 +0·64	−0·28 +0·24	
17 13	428	...	0·05	0·264	8	30	437	1¼	0·330	8·643 8·973	+0·64 +0·97	+0·64 +0·97	
13 12	381	...	0·05	0·264	8	25	367	1	0·264	8·973 9·237	+0·97 +1·24	+0·97 +1·24	
13 10	...	...	...	...	...	...	...	...	...	...	...	...	

(1) The actual rate of run-off will be ·406 inches.

APPENDIX IV.

Statement showing levels and sizes of Channels to suit ⅞"(1) inch run-off, with 3' depth over weir, and cut-off at 7' and 10·50 flood-level in swamps.

Channel lettered.	Required discharge.	Level of surface.	Longitudinal slope of surface. Per 1,000.	Feet per mile.	Depth.	Bed width.	Calculated discharge.	Length.	Total surface fall.	Resulting surface level.	Resulting bed level.	Selected bed level.
1	2	3	4	5	6	7	8	9	10	11	12	13
					Ft.	Ft.						
Diversion ...	5,115	3·00	0·20	1·056	10	130	5,140	⅝	0·000	3·000 3·800	− 7·00 − 0·34	− 7·00 − 7·00
0 ... 0_1 ...	5,115	...	0·05	0·264	10 11	210 200	5,160 5,020	1⅜	0·300	3·800 4·056	− 0·34 − 6·94	− 7·00 − 7·00
0_1 ... 20 Bendal ...	5,115	...	0·05	0·264	11 11	200 200	5,020 5,020	3⅜	0·924	4·056 4·980	− 6·94 − 0·02	− 7·00 − 6·00
20 Bendal ... 5 Usti ...	4,644	...	0·10	0·528	11 12	200 130	5,020 over 4,611	3¾	1·980	4·980 6·960	− 6·02 − 5·04	− 6·00 − 5·00
4 Usti ... 5 Nainan ...	2,659	10·50	0·10	0·528	11	80	2,700	6½	3·432	6·960 10·392	− 4·04 − 0·61	− 4·00 − 1·00
5 Usti ... 10 Sangrampur ...	1,190	0·060	0·05	0·264	10	55	1,160	2½	0·660	6·960 7·620	− 3·04 − 2·38	− 3·00 − 2·34
10 ... 18 ...	1,008	...	0·05	0·264	10	50	1,003	4	1·056	7·620 8·670	− 2·38 − 1·32	− 2·34 − 1·28
18 ... 17 ...	720	...	0·05	0·264	9	40	700	2	0·528	8·676 9·204	− 0·32 + 0·20	− 0·28 + 0·24
17 ... 1 ...	515	...	0·05	0·264	8	37·5	541	1¼	0·330	9·204 9·534	+ 1·20 + 1·53	+ 0·64 + 0·97
13 ... 12 ...	494	...	0·05	0·264	8	35	500	1	0·264	9·534 9·798	+ 1·53 + 1·80	+ 0·97 + 1·24
13 ... 10 ...	...	...	...	...	...	30	...	...	..	...	...	+ 0·00 + 0·00
5 ... 1 ...	565	+ 6·000	0·10	0·528	8	25	505	6	3·168	0·960 10·128	− 1·040 + 2·128	− 1·04 + 2·13

(1) The actual rate of run-off will be ·009 inches.

APPENDIX V.

Statement showing levels and sizes of inland channels suited to run-off of ½".

Channel lettered.	Required discharge.	Level of surface.	Longitudinal slope of bed. Per 1,000.	Feet per mile.	Depth.	Bed width.	Calculated discharge.	Length.	Total surface fall.	Resulting surface level.	Resulting bed level.	Selected bed level.	Remarks.
1	2	3	4	5	6	7	8	9	10	11	12	13	14
					Ft.	Ft.							
4 ... ... 7	444	+6·40	0·05	0·264	7	40	455	5	1·320	6·400 7·720	−0·600 +0·720	−0·60 +0·72	
8 ... ... 7	84	...	0·05	0·264	5·5	12	over 85	6	1·584	9·304 7·720	+3·304 +2·220	+2·00 +2·00	
7 ... ... 6_1	252	...	0·05	0·264	6	32	278	3	0·792	7·720 8·512	+1·721 +2·512	+1·72 +2·00	
6_1 ... ... 6	161	...	0·05	0·264	6	20	170	4	1·056	8·012 9·568	+2·012 +3·568	+2·00 +2·00	Connecting channel.
4 ... ... 10	182	...	0·05	0·264	6	30	201	2	0·528	6·400 6·928	+0·400 +0·928	+0·00 +0·00	
10 ... ... 9	168	...	0·05	0·264	6	20	170	7	1·848	6·928 8·770	+0·928 +2·770	+1·00 +1·00	
12 ... ... 14	84	6·904	0·05	0·264	6	14	90	5	1·320	... 8·224	+1·904 +3·224	+2·00 +2·00	Surface at (6)+4·09 slope 11 miles at ·264 per mile, fall 2·904 feet,
12 ... ... 11	67	6·904	0·05	0·264	5	12	73	4	1·056	... 7·960	+1·904 +2·960	+2·00 +2·00	
17 ... ... 15	24	6·310	0·05	0·264	5	10	67	3	0·792	... 7·102	+1·310 +2·102	+2·00 +2·00	
5 .. ... 1	183	+4·00	0·10	0·528	5	25	221	6	3·168	+4·000 +7·168	−1·000 +2·168	−1·04 +2·13	

APPENDIX VI.

Statement showing dimensions and cost of channels as designed.

Channel.	Bed Level. At tail.	Bed Level. At head.	Bed Level. Mean.	Mean level of ground or khal.	Mean depth of excavation.	Bed Width. At tail.	Bed Width. At head.	Bed Width. Mean.	Mean cross section of excavation.	Per Mile. Quantity of excavation.	Per Mile. Rate per 1,000.	Per Mile. Cost.	Length.	Total cost of earth-work.	Remarks.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
					Ft.		Ft.	Ft.		C.ft.	Rs.	Rs.	Miles.	Rs.	Rs.
Diversion ...	−7·00	−7·00	−7·00	+11·00	18·00	130	130	130	2,634	14,060,000	6	84,310	5/8	53, 48	Diversion ... 52,748
0—0₁	−7·00	−7·00	−7·00	−0·25	0·75	240	200	220	166	877,000	5	4,385	1½	6,578	Creek to Beodal ... 53,338
0₁—20 Bendal	−7·00	−0·00	−6·50	−1·00	2·50	200	200	200	506	2,672,000	5	13,360	3½	46,760	
20 (Bendal) to 5 (Usti).	−6·00	−5·00	−5·50	+5·00	10·50	200	130	105	1,811	9,721,000	5	48,605	3¾	1,84,120	Creek, Beodal to Usti ... 1,84,420
5 (Usti) to 4 (Noinon).	−4·00	−1·00	−2·50	+7·00	9·50	80	80	80	850	4,488,000	5	22,440	6½	1,45,860	
5—16 Nozra khal.	−3·00	−2·34	−2·07	+8·75	11·42	55	55	55	759	4,007,500	5	20,030	2½	50,008	
16—16	−3·34	−1·28	−1·81	+7·30	9·11	50	50	50	538	2,840,600	5	14,203	4	56,812	
18—17	−0·28	+0·24	−0·02	+7·00	7·02	40	40	40	320	1,737,100	5	8,680	2	17,372	
17—18	+0·04	+0·97	+0·80	+3·75	4·05	37·5	37·5	37·5	213	1,124,700	4	4,400	1¼	5,634	
18—13	+0·97	+1·24	+1·10	+5·00	3·90	35	35	35	150	824,000	4	3,296	1	3,290	
13—10	+0·00	+0·00	+0·00	+5·00	5·00	30	30	30	175	924,000	4	3,696	2	7,392	

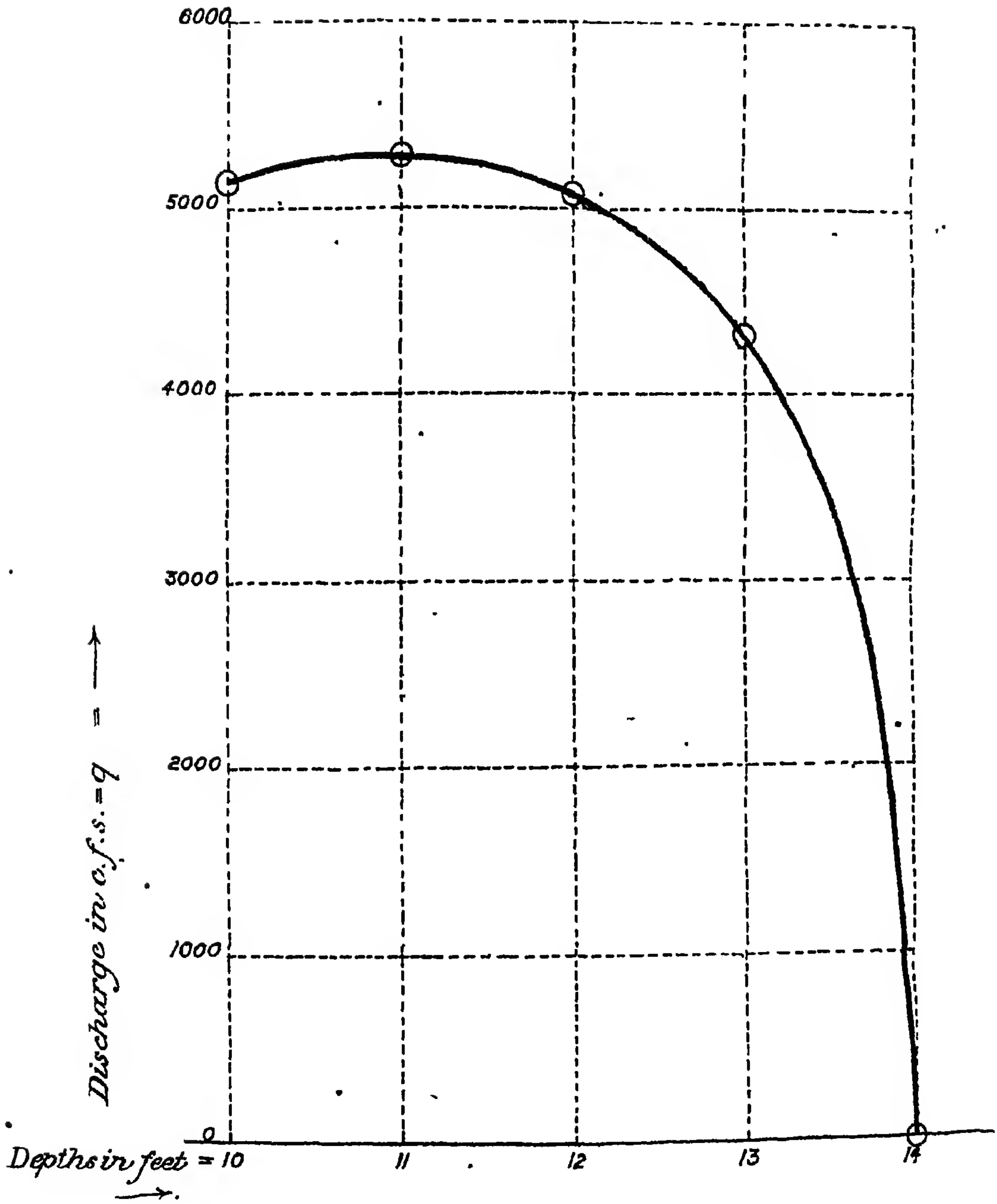
APPENDIX VII.

Statement showing dimensions and cost of channels as designed.

Channel.	Bed Level. At tail.	Bed Level. At head.	Bed Level. Mean.	Mean level of ground or khal.	Mean depth of excavation.	Bed Width. At tail.	Bed Width. At head.	Bed Width. Mean.	Mean cross section of excavation.	Per Mile. Quantity of excavation.	Per Mile. Rate per 1,000.	Per Mile. Cost.	Length.	Total cost of earth-work.	Remarks.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
										C. ft.	Rs.	Rs.		Rs.	Rs.
4—7	−0·60	+0·72	+0·00	+5·00	4·94	40	32	30	202	1,063,500	4	4,260	5	21,330	
6—7	+2·00	+2·00	+2·00	+5·00	3·00	13	12	12	45	237,000	...	951	6	5,706	
7—0₁	+1·72	+2·00	+1·86	+5·00	3·14	32	20	20	92	485,760	...	1,013	3	5,829	
0₁—0	+2·00	+2·00	+2·00	+5·00	3·00	20	20	20	69	364,320	...	1,457	4	5,828	
4—10	+0·00	+0·00	+0·00	+5·00	5·00	30	30	30	175	924,000	...	3,696	2	7,392	
10—0	+1·00	+1·00	+1·000	+3·40	2·10	20	20	20	54	285,120	...	1,140	7	7,980	
12—14	+2·00	+2·00	+2·00	+5·00	3·00	14	14	14	95	501,600	...	2,006	5	10,030	
12—11	+2·00	+2·00	+2·00	+5·00	3·00	12	12	13	85	448,800	...	1,795	4	7,100	
17—15	+2·00	+2·00	+2·00	+5·75	3·75	10	10	10	90	475,200	...	1,901	3	5,703	
5—1	−1·04	+2·13	+0·55	+8·00	7·45	25	5	15	167	881,760	...	3,527	6	21,162	93,120

APPENDIX VIII.

Curve of Channel Discharges



Scale 1/10

APPENDIX IX.

The figures in the body of the table are values of F, where

$$F=\frac{2}{3}\,\frac{\left(1-\frac{y}{x}\right)^{\frac{1}{2}}\left(1+\frac{1}{2}\frac{y}{x}\right)-\left(1-\frac{D}{x}\right)^{\frac{3}{2}}}{\frac{D}{x}}$$

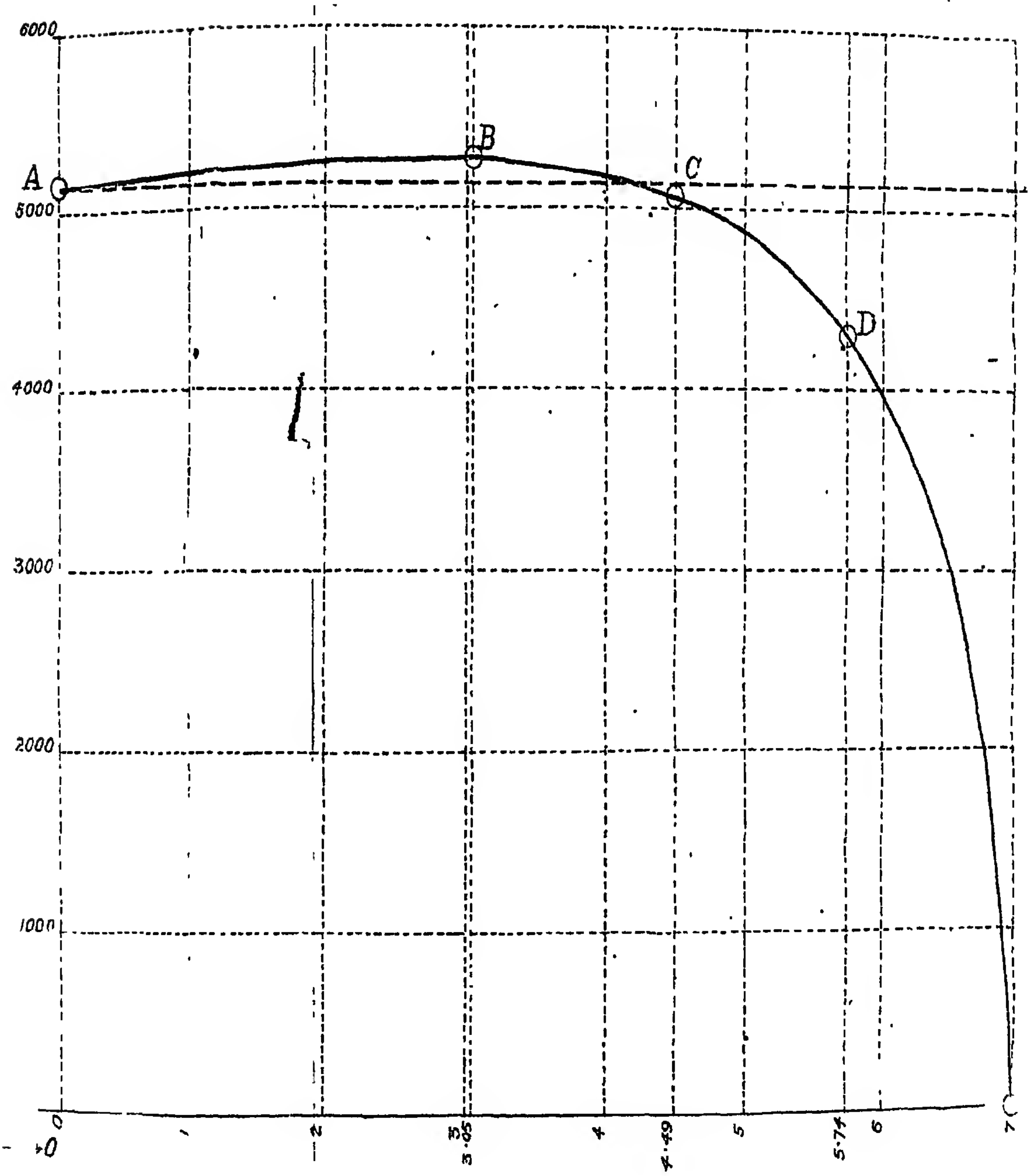
and $q=Fc\sqrt{2g}\,b\,Dx^{\frac{3}{2}}$

NOTE.— $\frac{y}{x}$ cannot exceed $\frac{D}{x}$. In case the actual value of y exceeds D, deduct the quantity $(y-D)$ from the actual values of both x and y, and use these reduced values for reference in the table.

$D/x=$	·01	·05	·1	·2	·3	·4	·5	·6	·7	·8	·9	·95	·99	1·00
y/x 0	·9978	...	·9740	·9482	·9207	·8921	·8619	·8309	·7968	·7583	·7173	·6939	...	·6667
·01	·995	...	...	...	...	...	...	...	...	...	...	...	...	...
·05	...	...	...	...	...	...	...	...	...	...	...	...	...	...
·1	...	...	·9487	·9353	·9121	·8850	·8507	·8257	·7922	·7550	·7145	·6912	...	·6641
·2	...	...	...	·8944	·8810	·8013	·8105	·8121	·7805	·7484	·7054	·6826	...	·6550
·3	...	...	...	...	·8367	·8200	·8115	·7850	·7509	·7273	·6843	·6671	...	·6414
·4	...	...	...	...	...	·7740	·7879	·7617	·7257	·7000	·6651	·6444	...	·6197
·5	...	...	...	...	...	...	·7071	·7010	·6858	·6621	·6318	·6124	...	·5893
·6	...	...	...	...	...	...	...	·6325	·6265	·6100	·5850	·5691	...	·5481
·7	...	...	...	...	...	...	...	...	·5477	·5417	·5243	·5111	...	·4929
·8	...	...	...	...	...	...	...	...	...	·4472	·4404	·4316	...	·4174
·9	...	...	...	...	...	...	...	...	...	...	·3162	·3130	...	·3057
·95	...	...	...	...	...	...	...	...	...	...	...	·2236	...	·2139
·99	...	...	...	...	...	...	...	...	...	...	...	...	·1000	·0997

APPENDIX. X.

Curve showing corresponding values of discharges (q) and tideway heights (y)



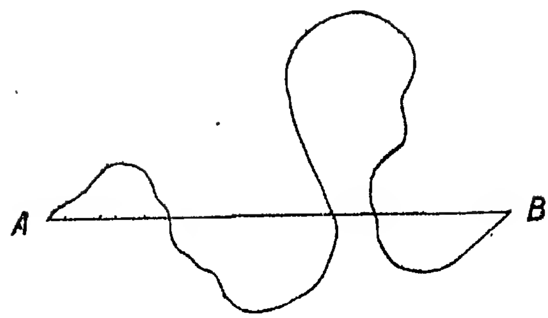
Scale 1/10

X.—ENQUIRIES INTO DRAINAGE SCHEMES.

I will, in conclusion, indicate briefly the main points which have to be enquired into when investigating schemes for flood-drainage. The nature and scope of the works will, of course, depend on the object to be attained, and it will be necessary to enquire what their object is, and what is the nature of the damage it is intended to remedy or prevent. If the object is to avert damage to standing crops during the rainy season, the works must be of sufficient capacity to carry off the rainfall in sufficient time to prevent the crop from being destroyed. It must be remembered that rice will, as a rule, bear being submerged for seven days without being killed. A certain amount of water, too, is held up by field-ridges, &c., and some is absorbed, so that it is not necessary to provide for carrying off the whole rainfall at the same rate that it falls. The usual allowance in Lower Bengal is to provide for a run-off of $\frac{3}{4}$ inch in 24 hours, over the whole of the catchment area. Do not make the mistake of calculating your run-off over only the low-lying part, which forms the swamp proper. If, on the other hand, the object is to reclaim land, not by draining it during the flood-season, but by drying it after the rains are over, so as to enable cold-weather crops or "*boro*" *dhan* to be sown, you will have to enquire from the cultivators by what date they require the land to be dried, and you will have to calculate the time in which your channels will achieve this object in the way I have indicated to you.

As regards the observations to be made, the whole locality should be carefully examined, so as to determine the general extent of flooding, the sources whence the water comes from, and the best direction in which to drain it off. The general direction of the slope of the land (if there is any slope) and the locality and direction of flow of the natural drainage channels (if such exist) should be determined. Having determined the direction in which the drainage should be conducted, the capacity of the outfall is of the first importance. If it is a river, its capacity should be ascertained by velocity observations, so as to determine whether it will be able to carry off the extra volume it is proposed to throw into it. If the outfall is tidal, a gauge should be erected, and tidal observations taken over as long a period as practicable during the time of year that drainage will be in progress. Even if the outfall is of sufficient capacity, its liability to silt up and thereby become useless must be most carefully considered. In case there are any rivers or streams coming into the area to be drained, the amount of water they bring in must be carefully determined by observations extending over the time of year that the drainage will have to be carried out. The respective levels of the outfall and of the area to be drained are absolutely essential, and must be very carefully ascertained. Lines of flying levels should be run across the area to be drained, to determine the general level, and the level of the lowest part. These lines should be sufficiently numerous, if practicable, to determine the area of the flooded tract at various levels, from full flood level down to the lowest level. It will not, as a rule, be possible to run complete contours, at any rate for a preliminary estimate, but in a finished scheme it may be necessary to do so, not only to determine the area but also for purposes of assessing the cost on the cultivators of lands situated at different levels; the low-level lands receiving, of course, more benefit from the scheme than the fields at higher levels. The area of the whole catchment area must also be ascertained. The existing flood-levels must be carefully determined, though they may, of course, be somewhat altered by the operation of the drainage works, when constructed, and you will have to take into consideration, in making your calculations, the effect which the drainage works will probably have on the flood-levels.

Fig: 1.


A
B

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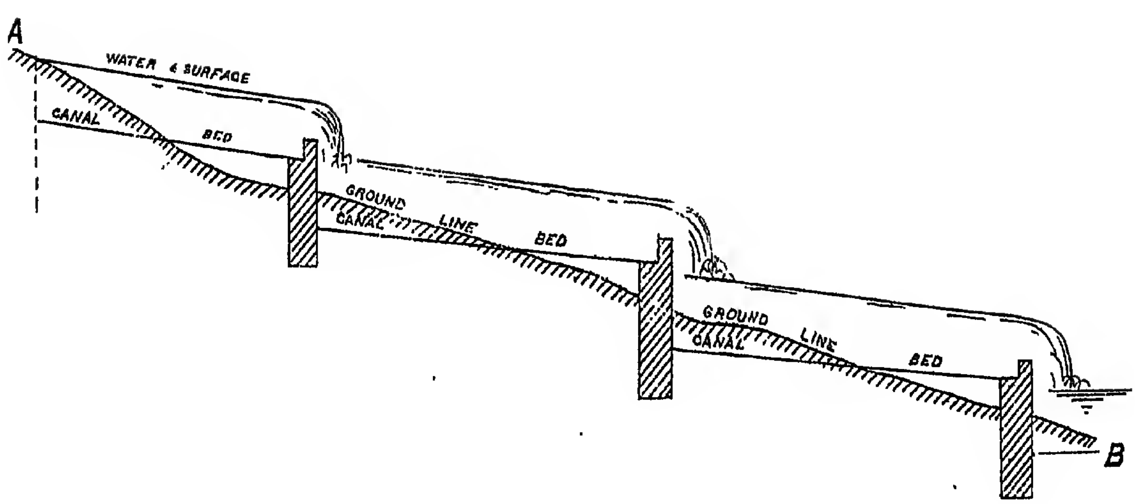
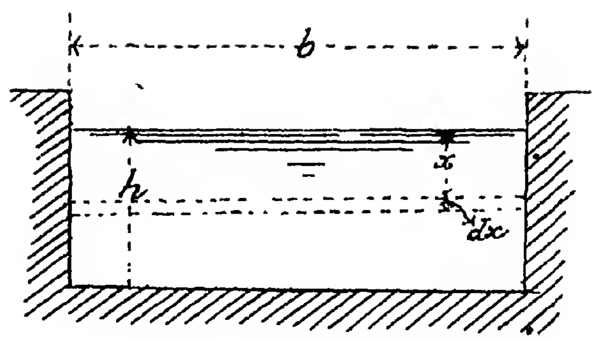

A
WATER & SURFACE
CANAL
BED
GROUND
CANAL
LINE
BED
GROUND
CANAL
LINE
BED
B

Fig: 3.



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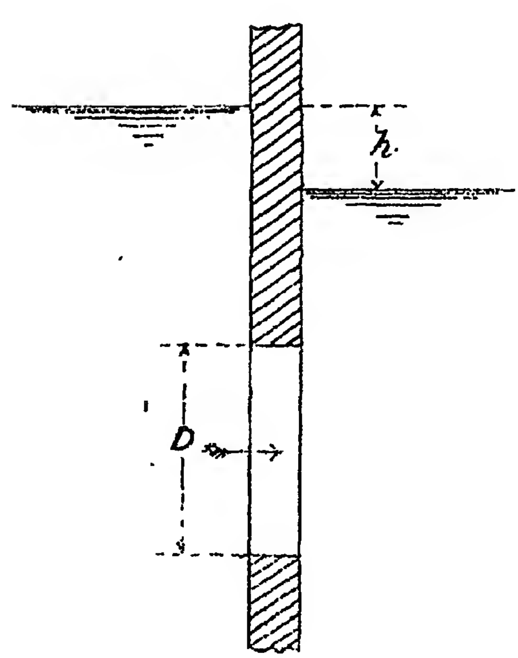

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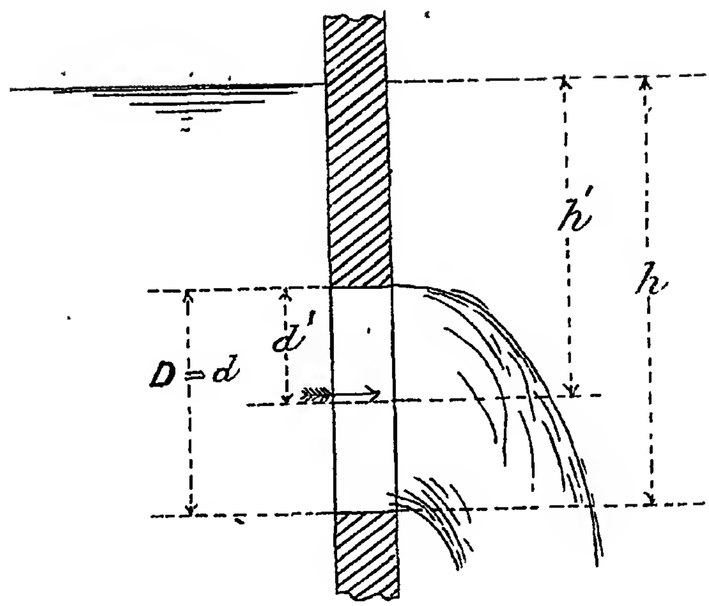

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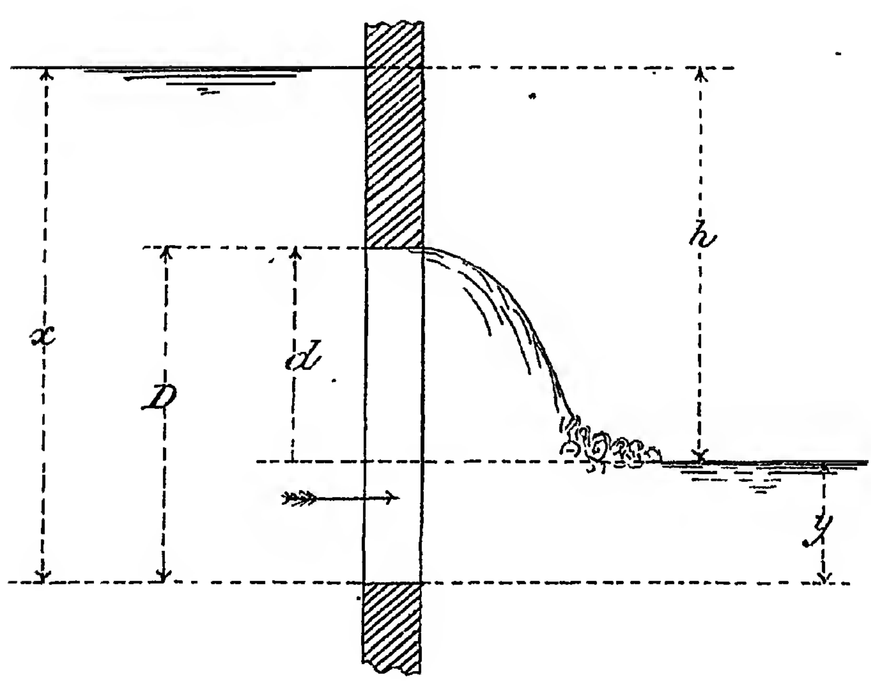

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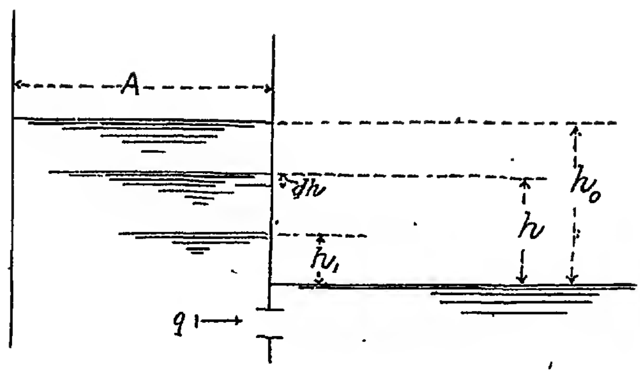

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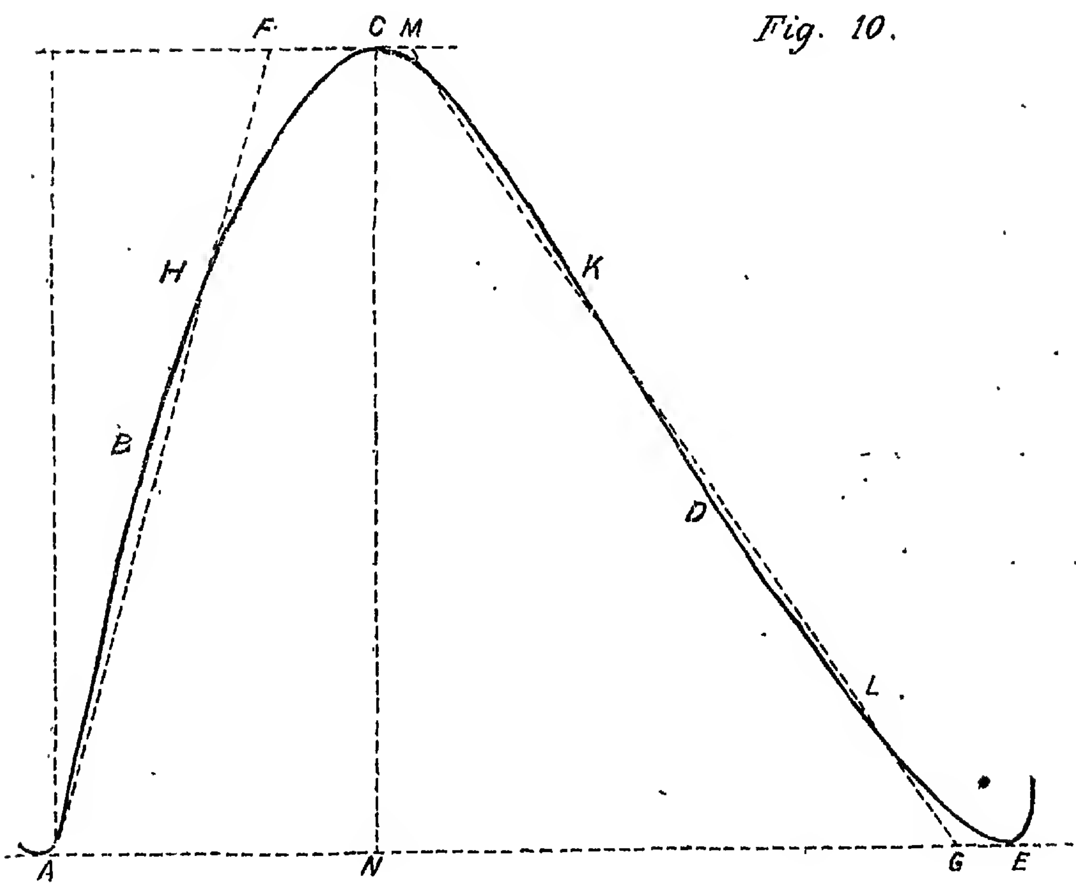

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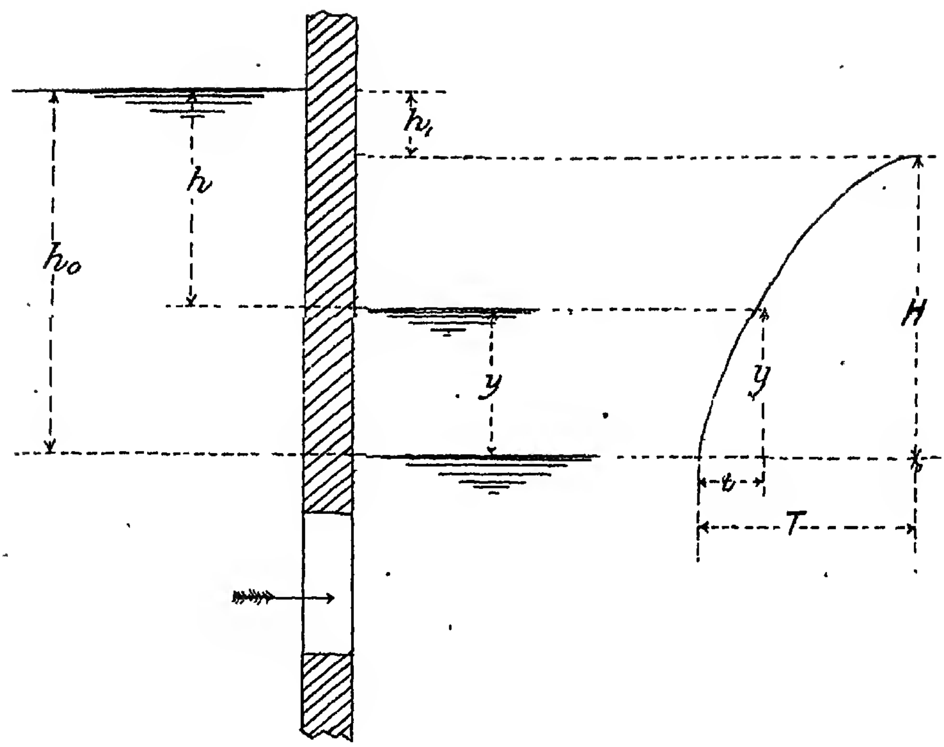

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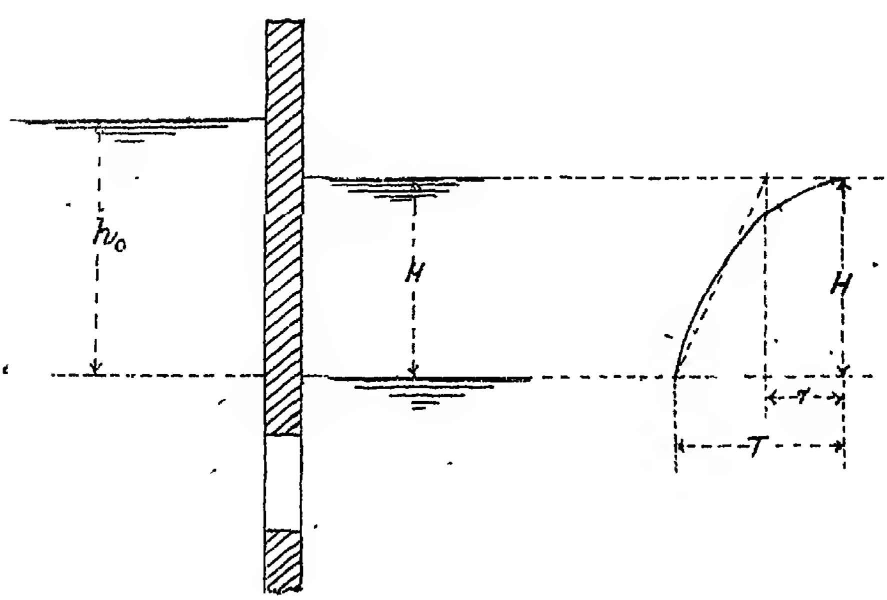
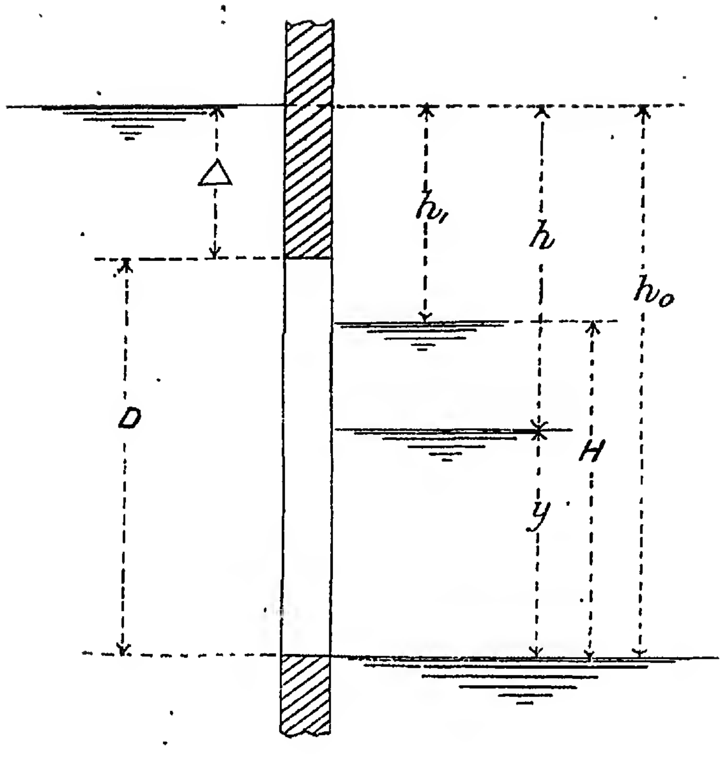


Fig: 15



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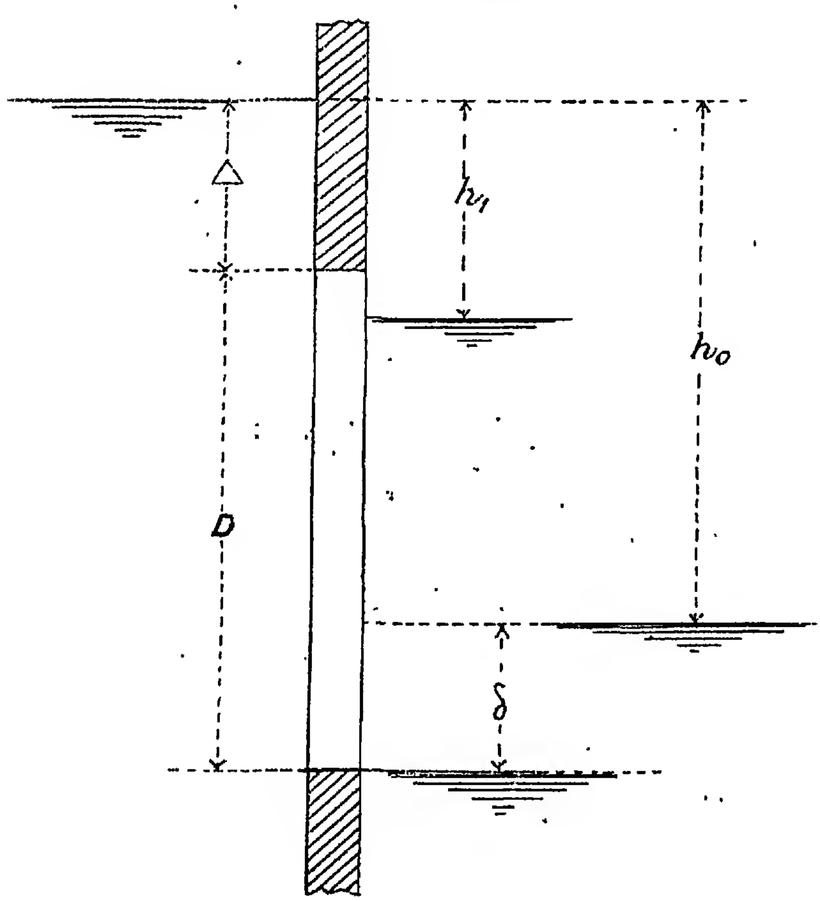

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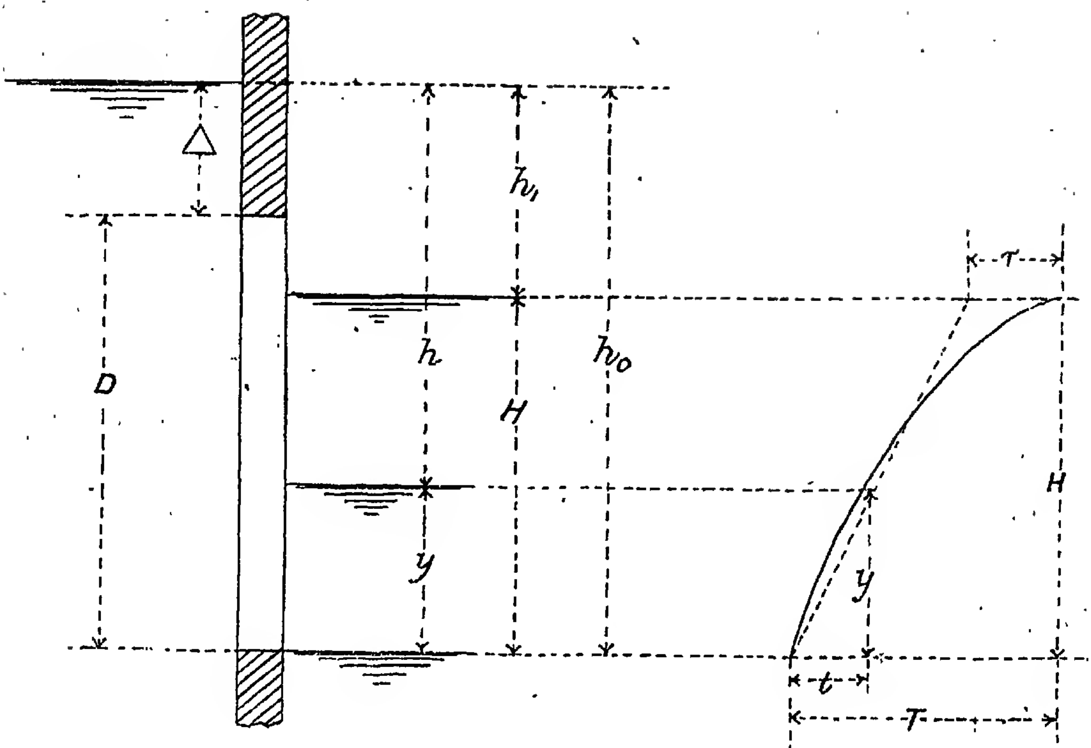

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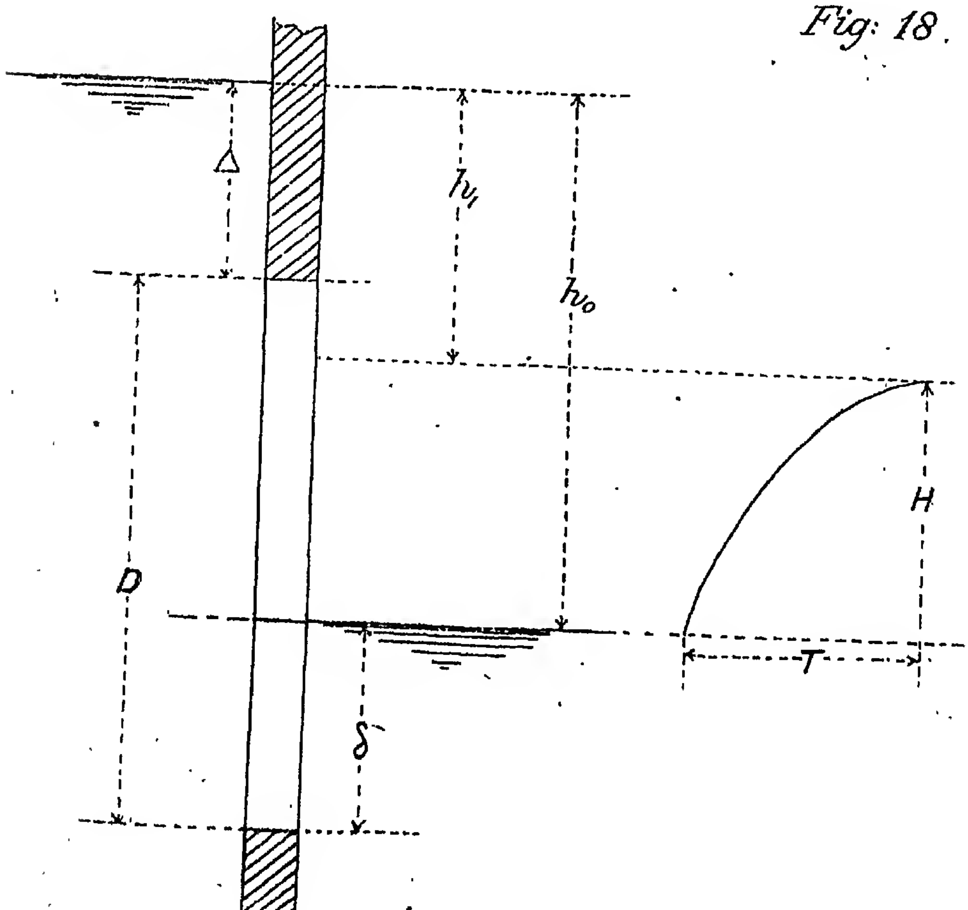

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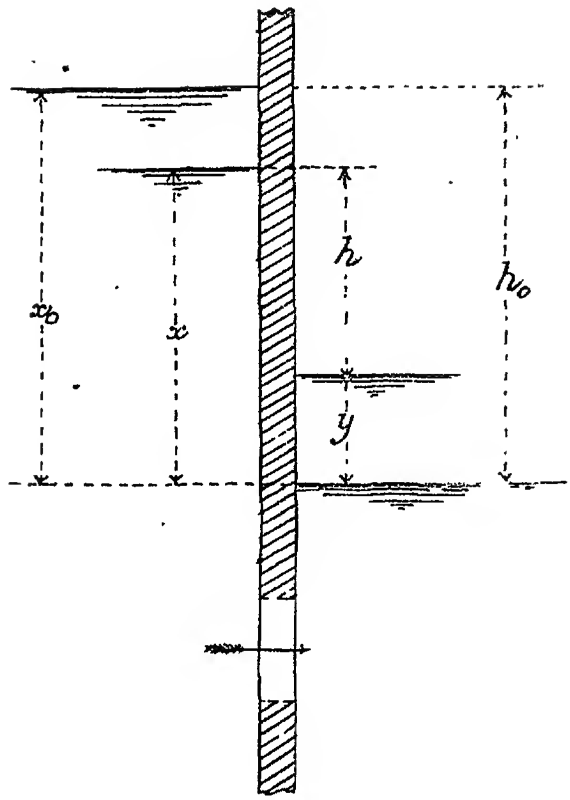

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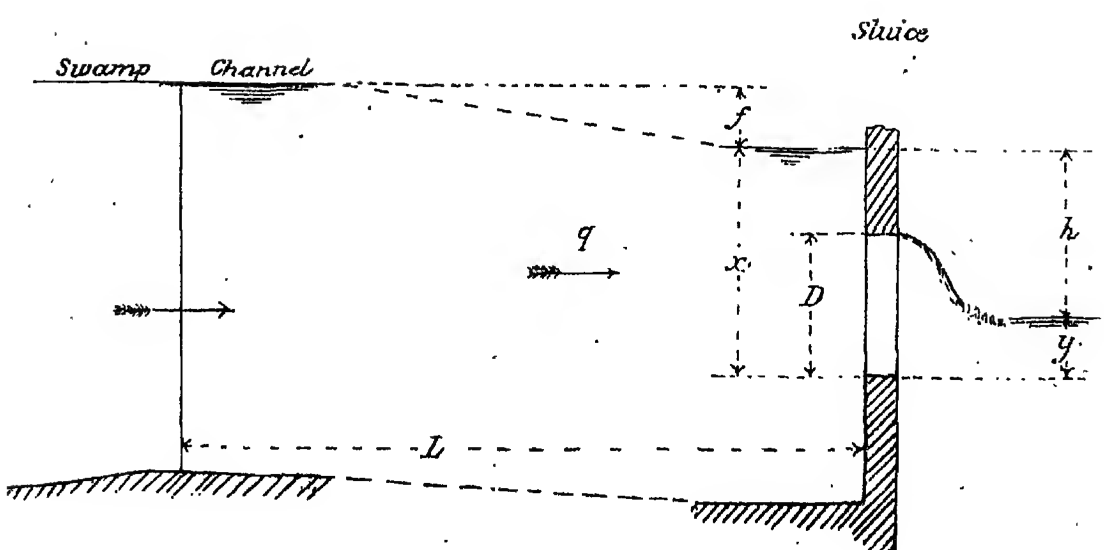

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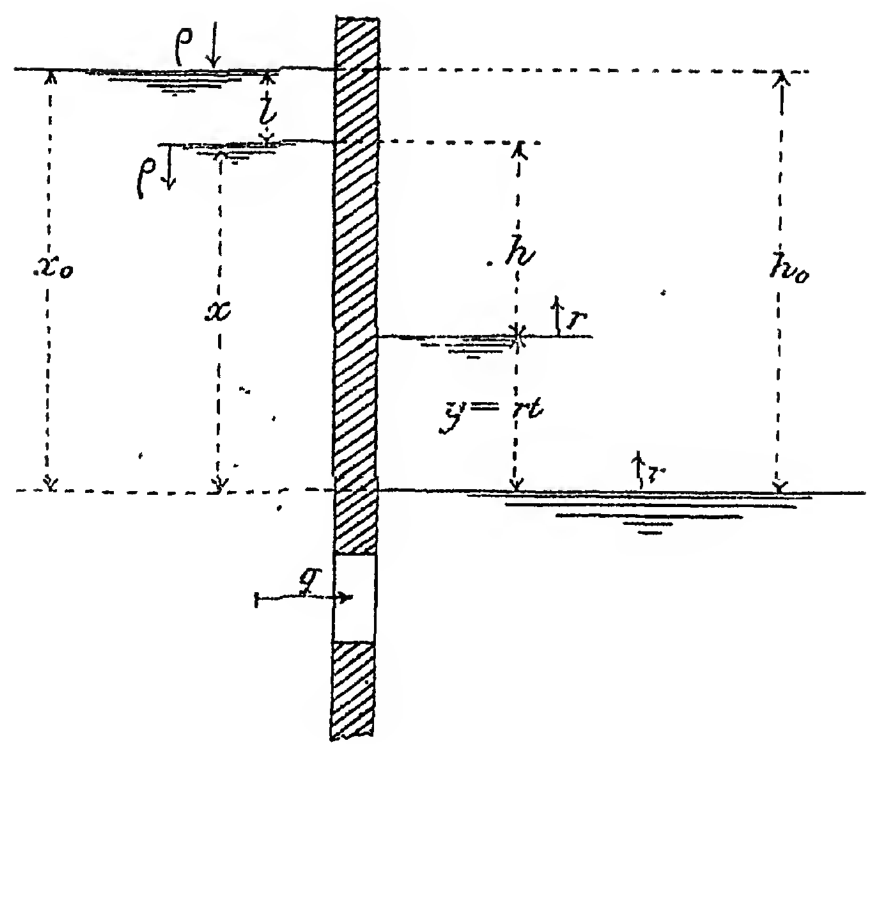

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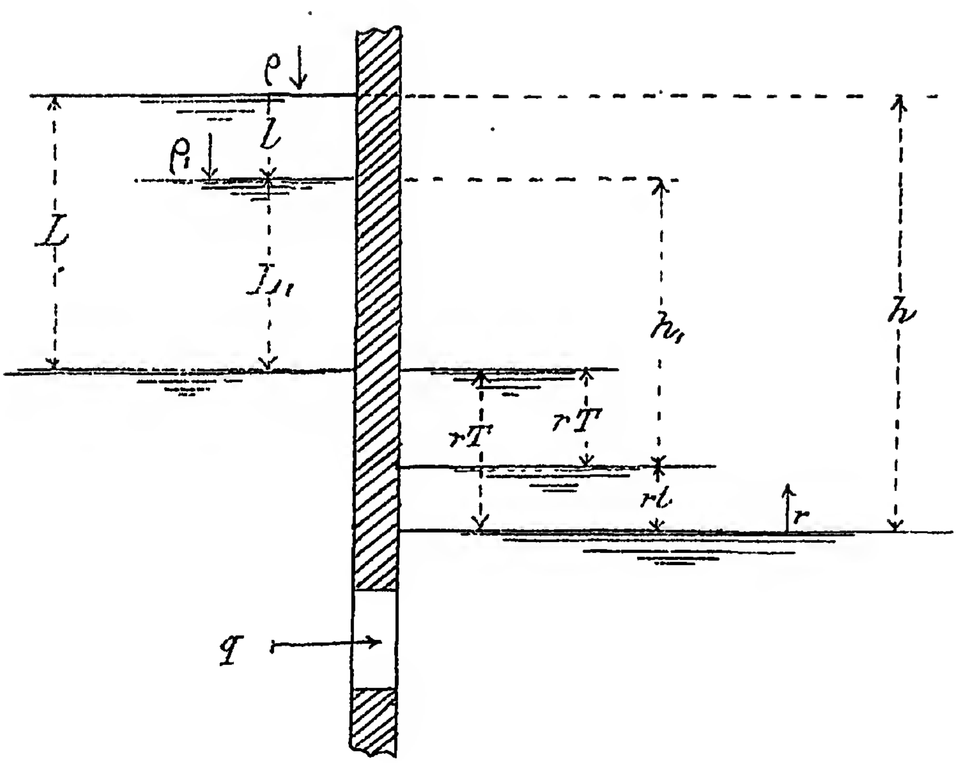

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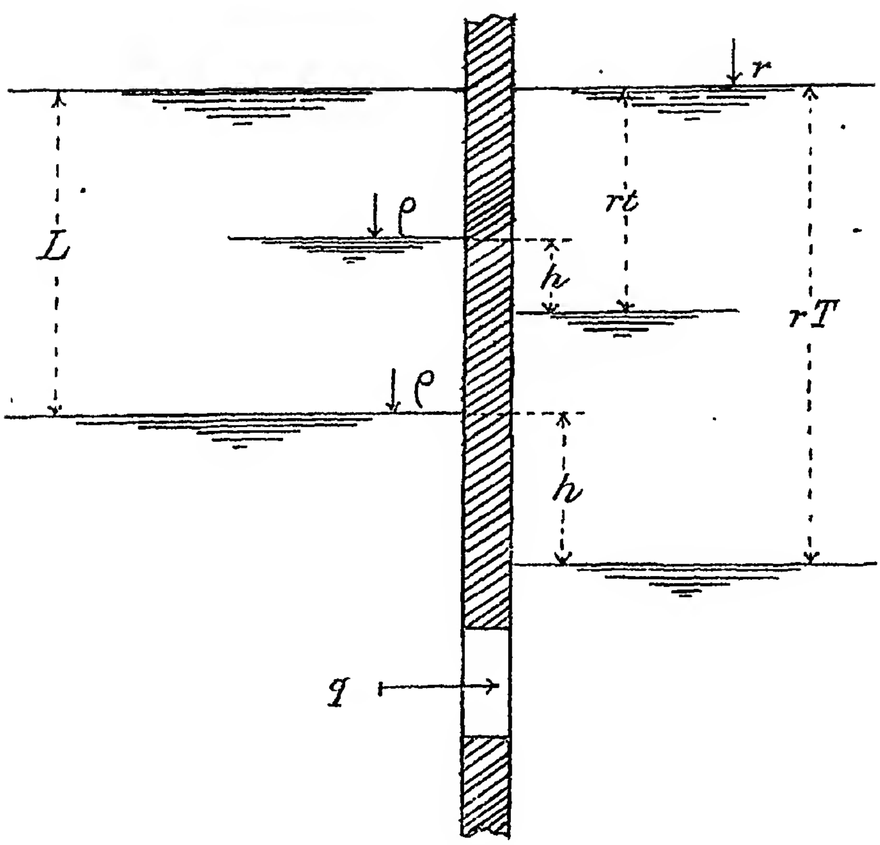

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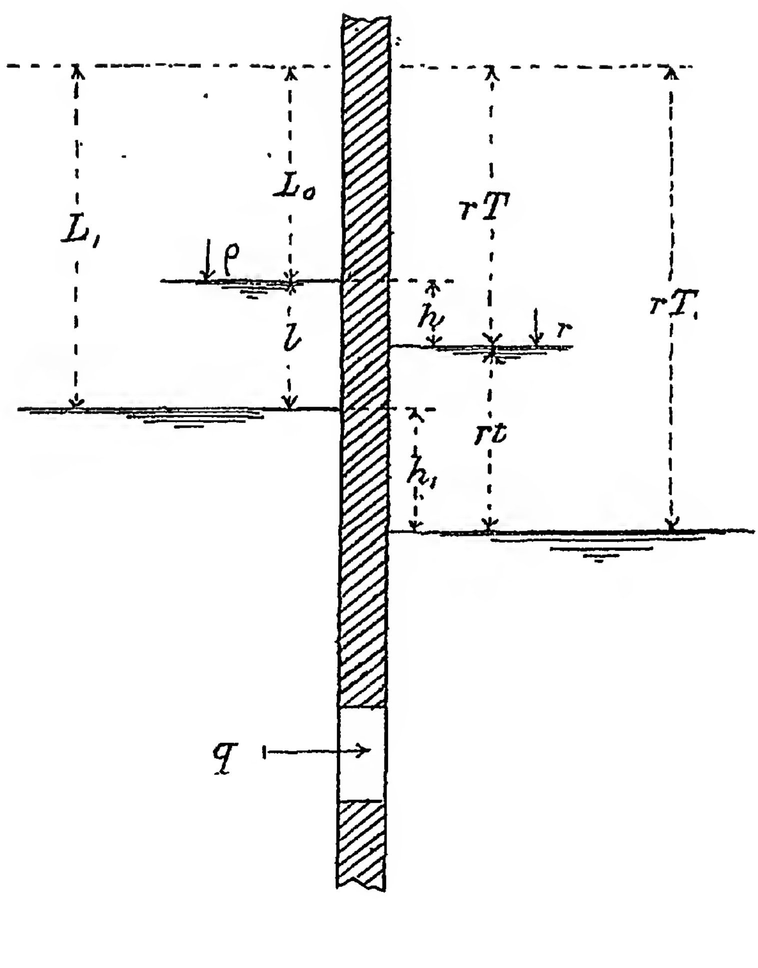

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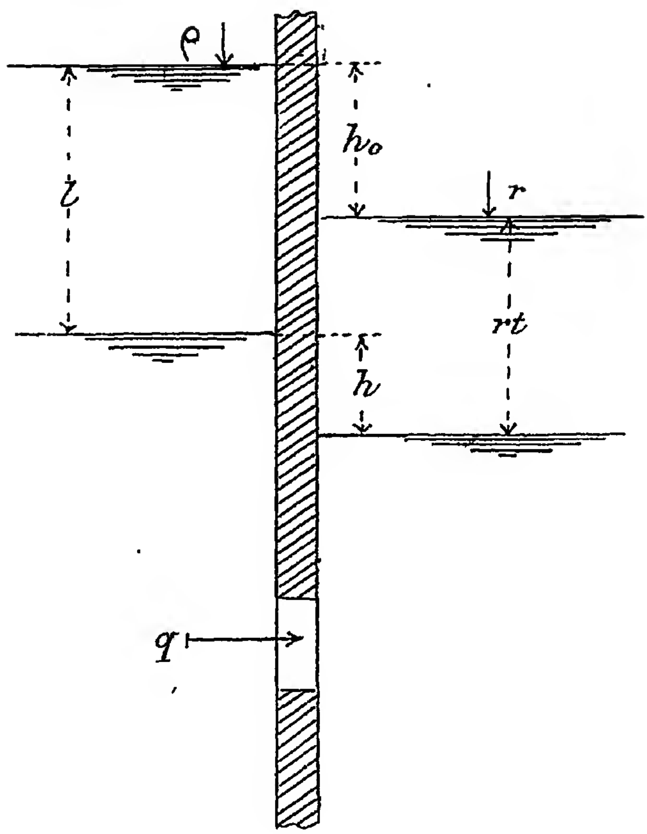

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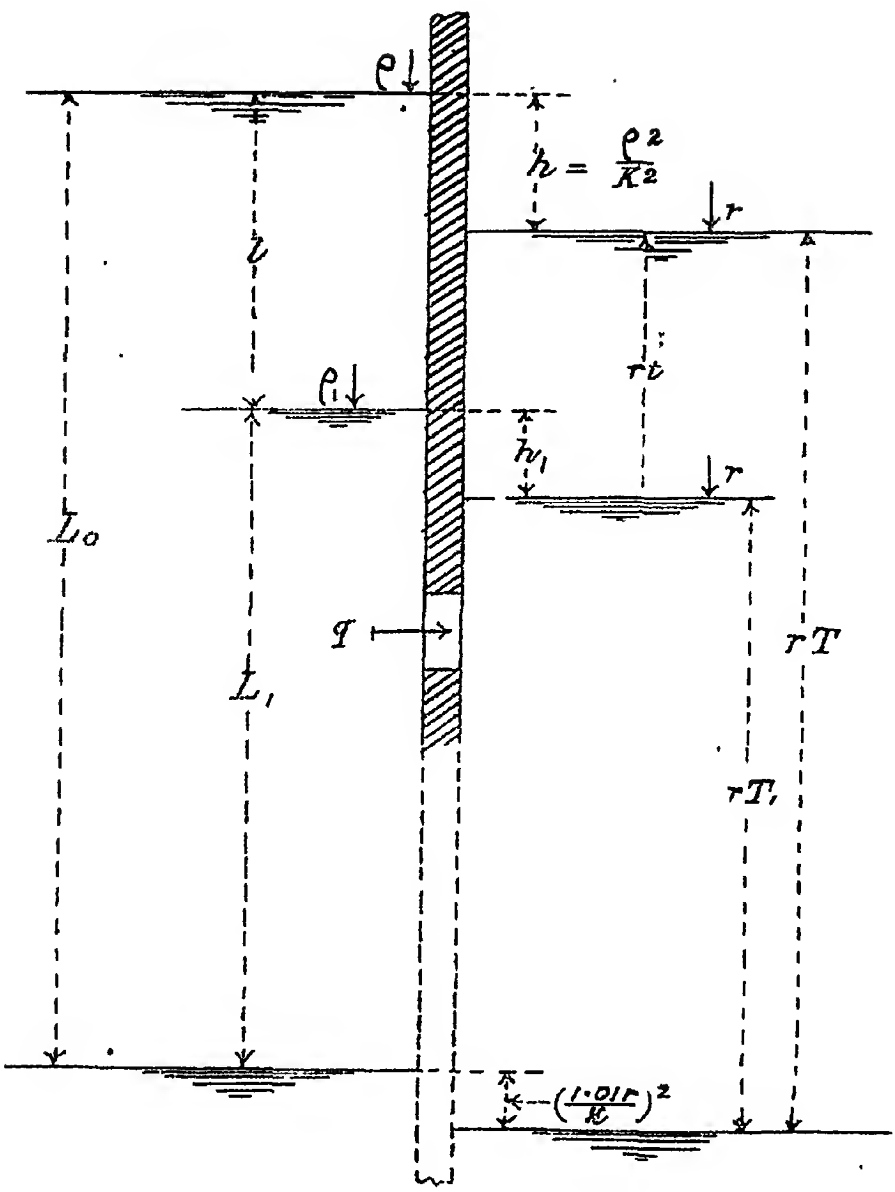

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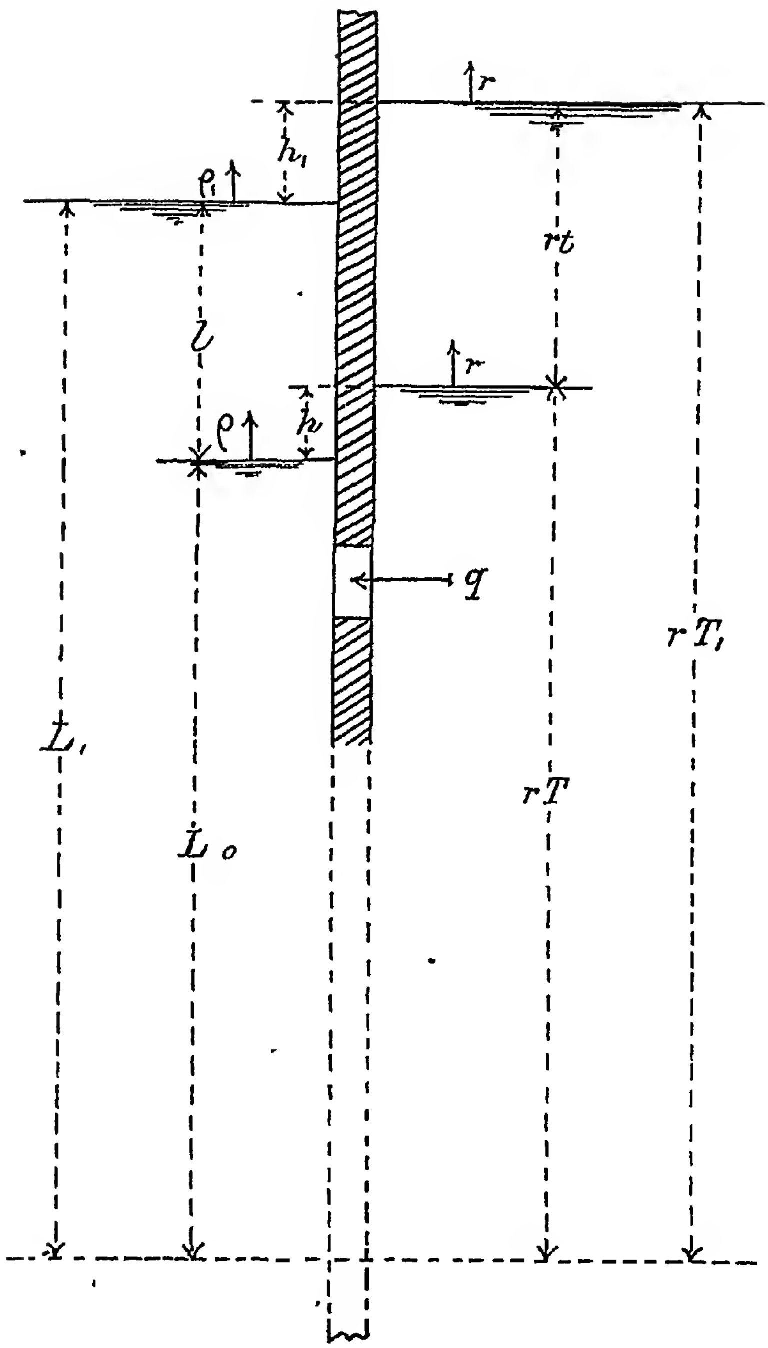

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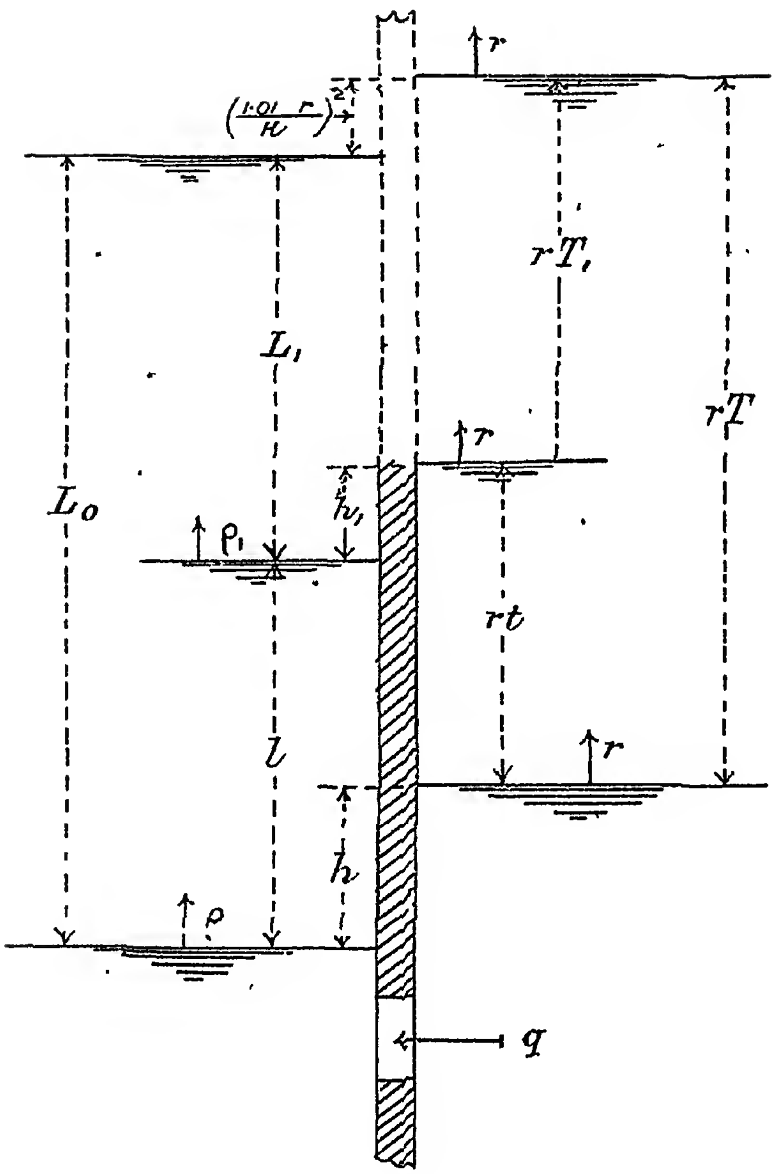
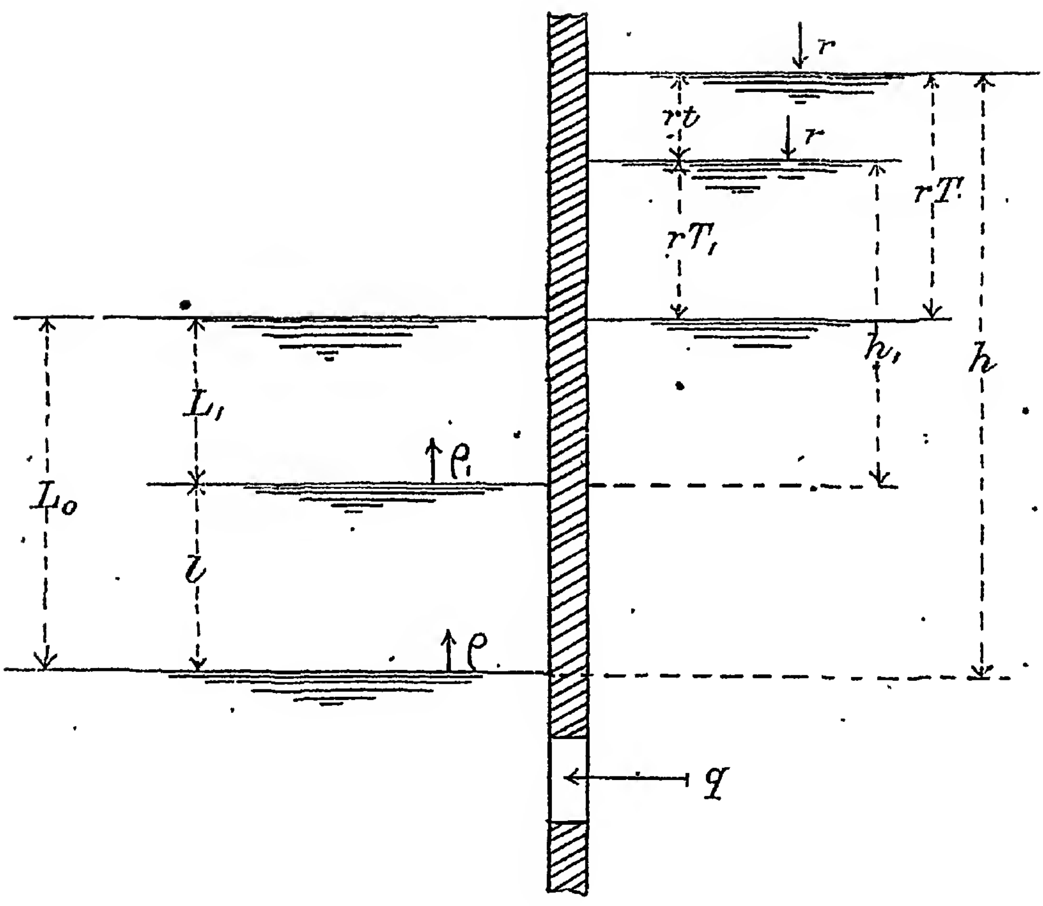


r
$\left(\frac{1{\cdot}01\ r}{\kappa}\right)^2$
L_1
rT_1
rT
L_0
r
h_1
ρ_1
rt
l
r
h
ρ
q

Fig: 29.



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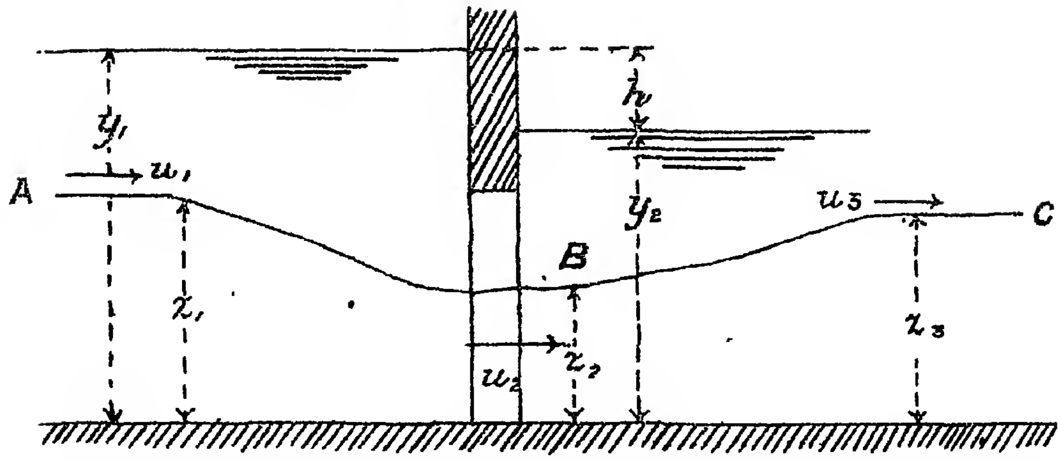

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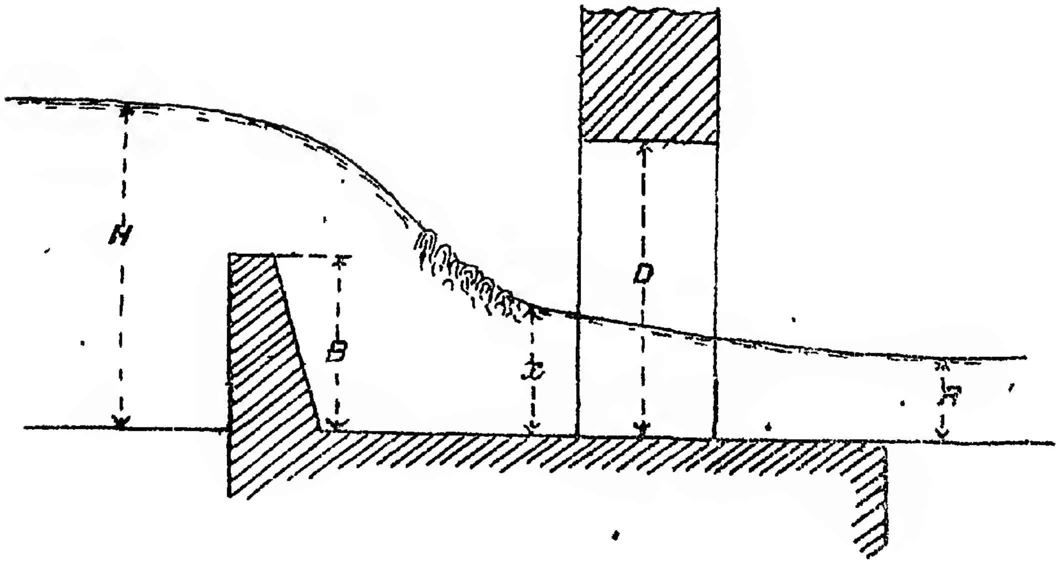

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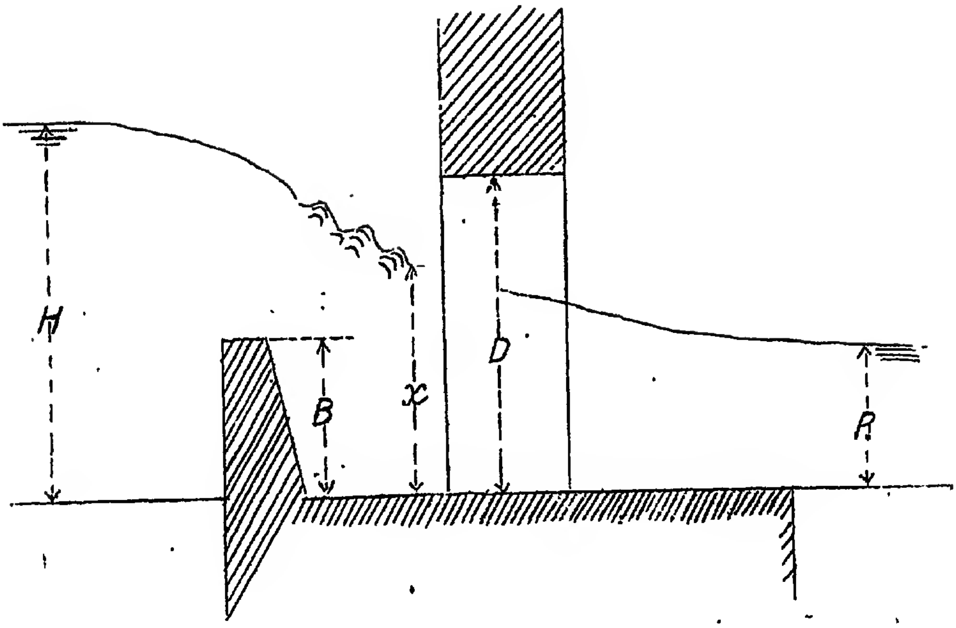

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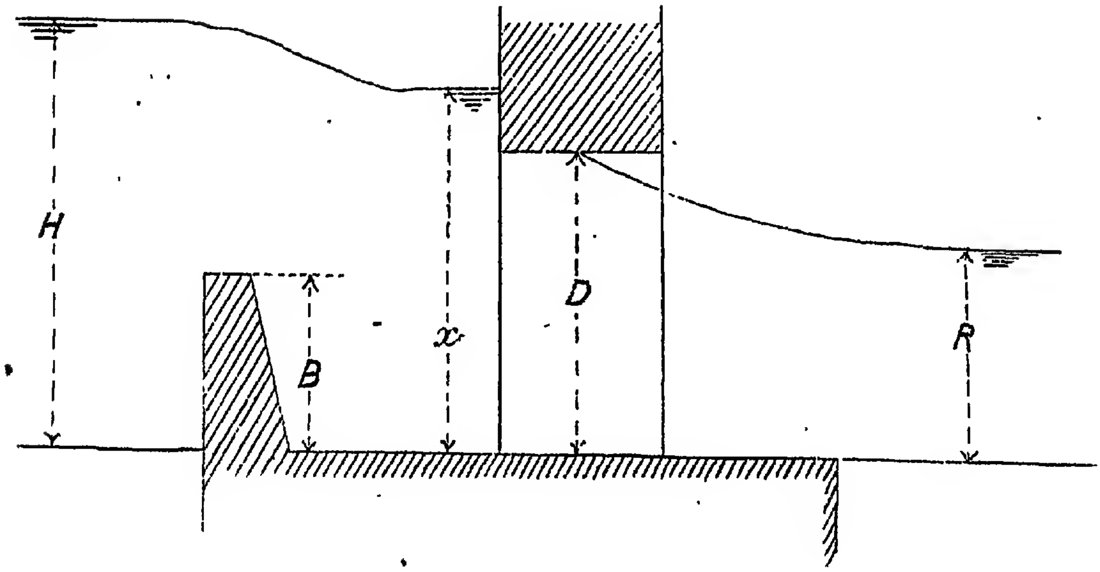

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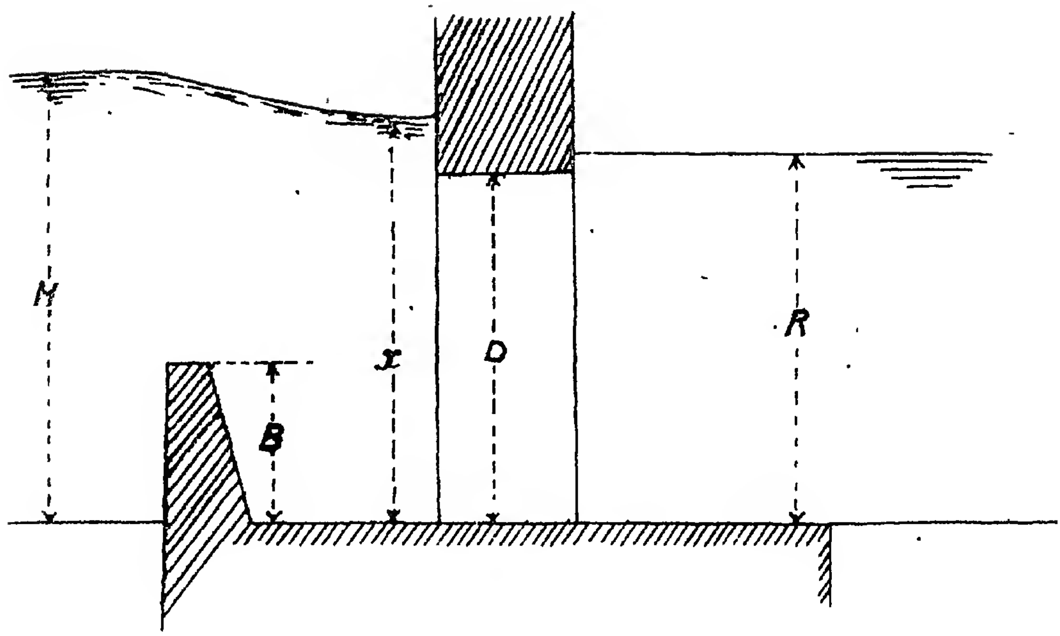

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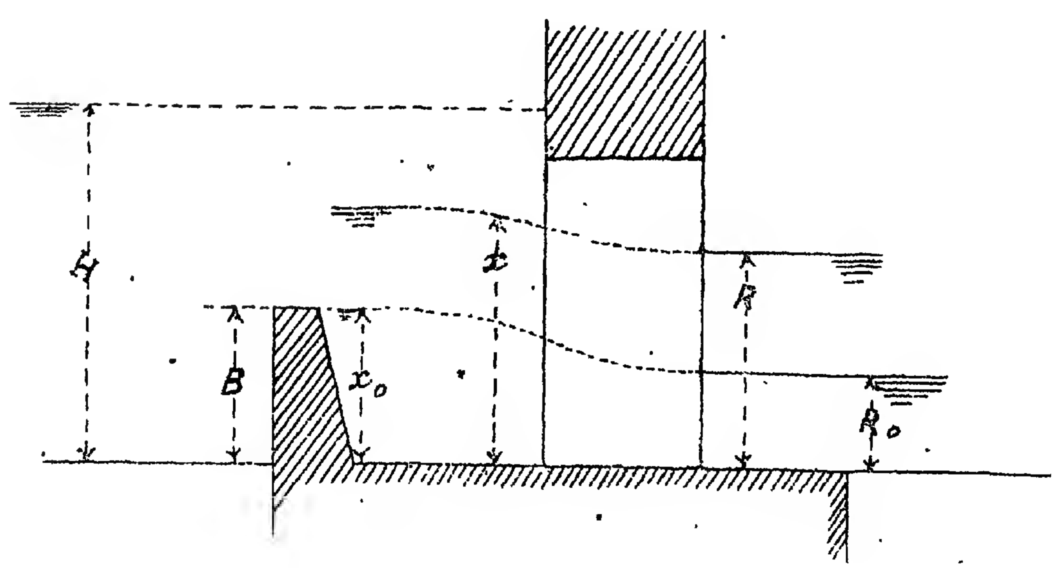

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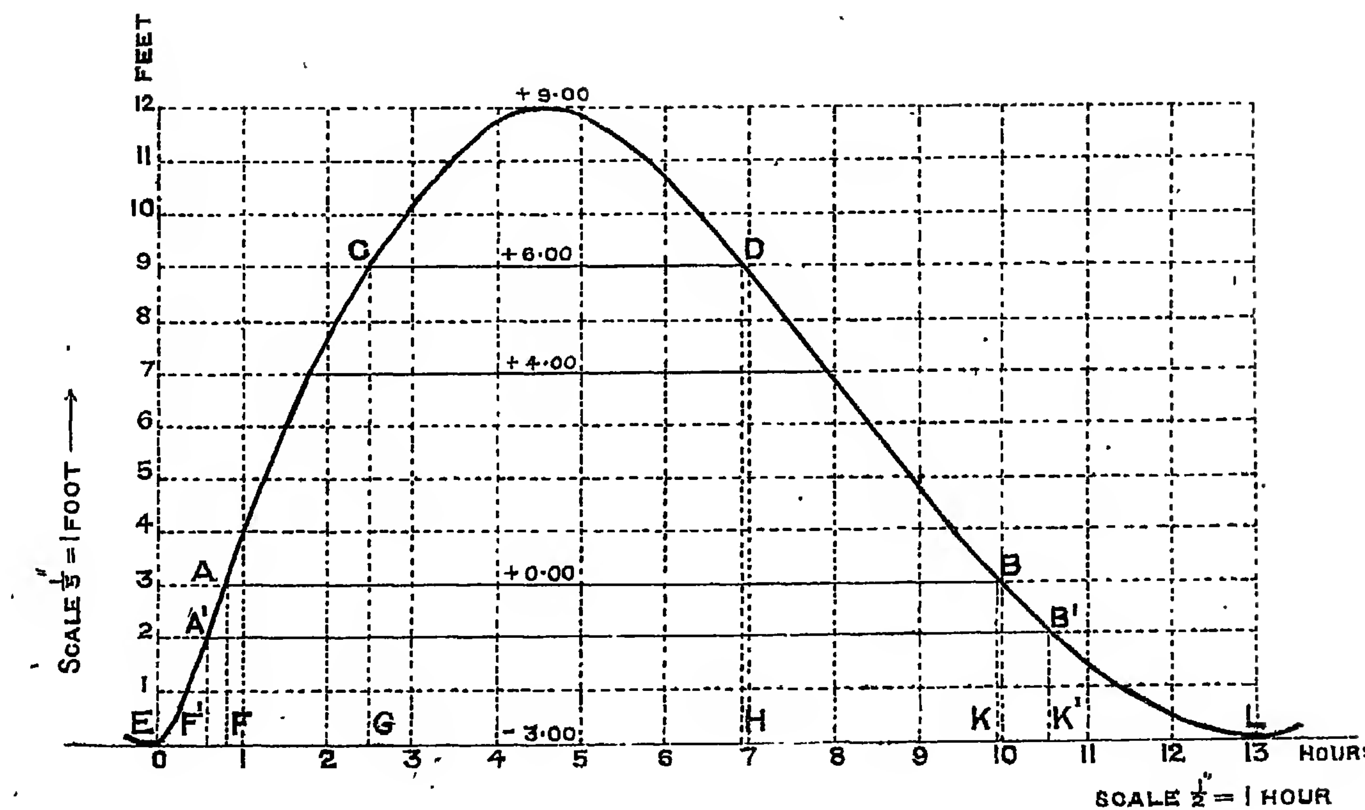
FEET
+ 9·00
+ 6·00
+ 4·00
+ 0·00
- 3·00
A
A'
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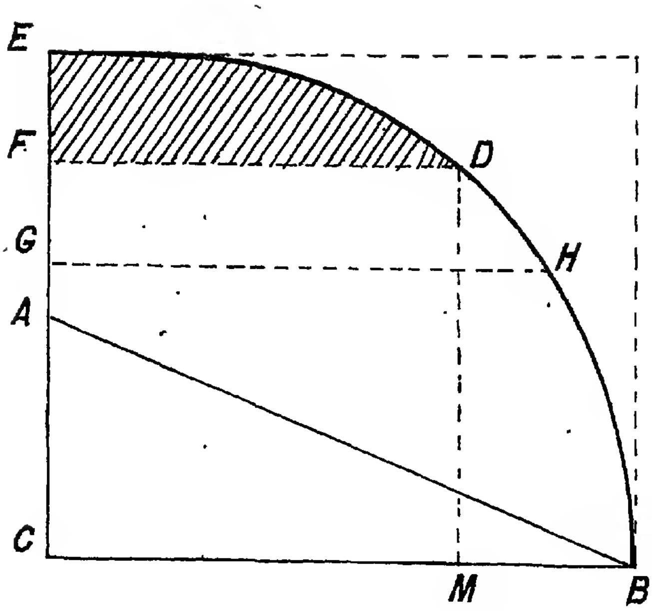
13
HOURS
SCALE 1/5" = 1 FOOT
SCALE 1/2" = 1 HOUR

Fig: 37.



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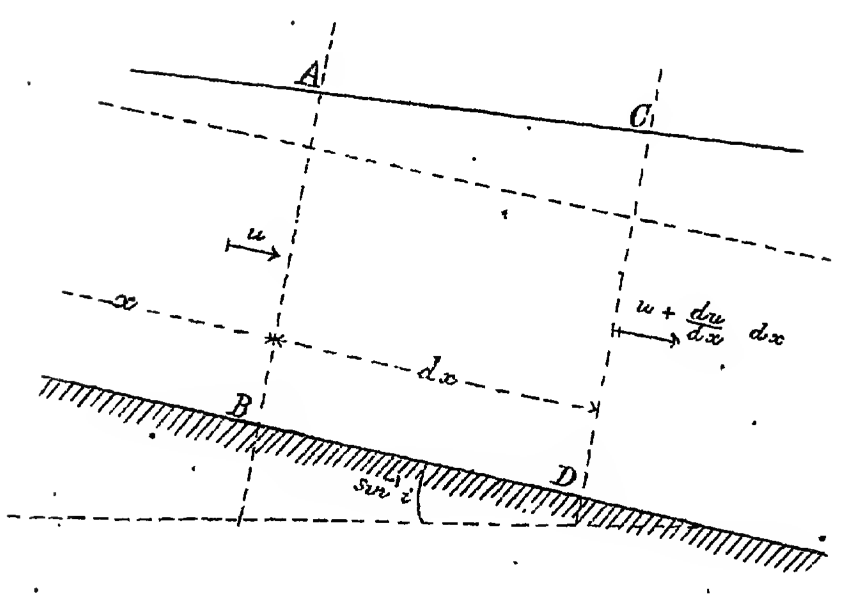

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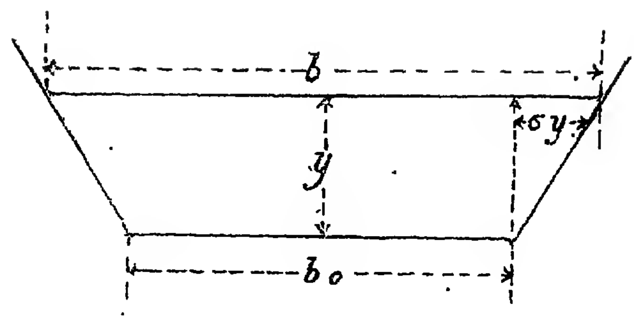

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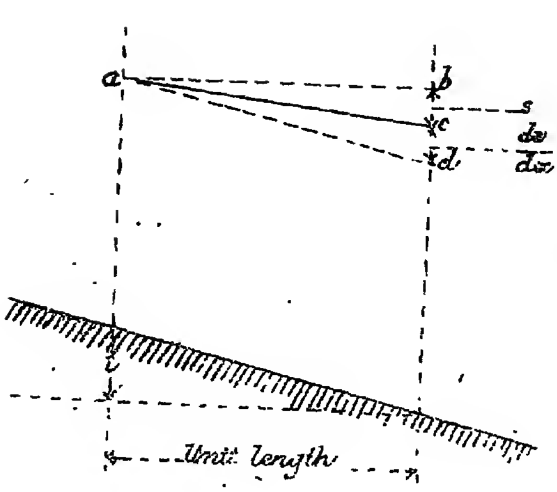

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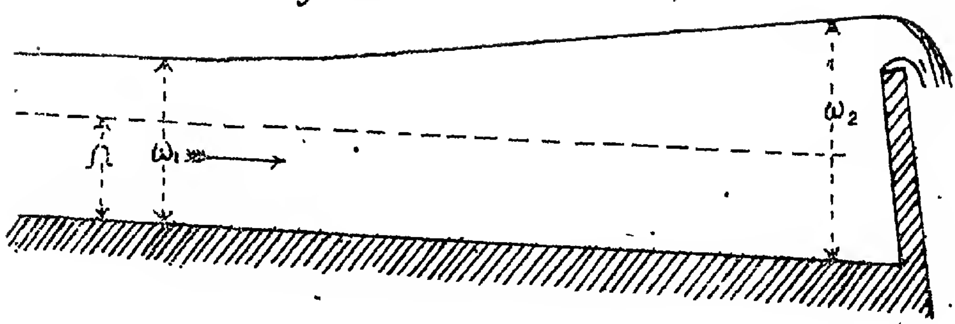

Fig: 42.